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Comparative Performance of Mamdani and Sugeno Fuzzy Inference Systems with Gaussian Membership Functions for Mathematics Learning Assessment

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ABSTRACT
Fuzzy inference systems (FISs) provide a framework for modelling the gradual and uncertain nature of student learning achievement. This study presents a controlled comparison of Mamdani and Sugeno fuzzy inference systems using Gaussian membership functions for mathematics learning assessment. The comparison retains the same dataset, input variables, Gaussian membership functions, and expert-defined rule base while changing only the inference mechanism. The dataset comprises 360 secondary school students, with examination score, assignment completion, and class participation used as input variables and teacher evaluation used as the benchmark for learning achievement. Both systems employ three Gaussian linguistic terms for each input and the same 27-rule knowledge base, while a zeroth-order Sugeno formulation is used for the Sugeno model. Model performance is evaluated using Pearson correlation coefficient, root mean square error (RMSE), and mean absolute error (MAE). Under the present dataset and model configuration, the Sugeno-Gaussian FIS achieved a Pearson correlation of 0.98996, RMSE of 2.7651, and MAE of 2.4279, compared with 0.9623, 7.2630, and 6.5607, respectively, for the Mamdani-Gaussian FIS. The results indicate that the Sugeno-Gaussian model achieved stronger linear agreement with teacher evaluations and lower prediction errors than the Mamdani-Gaussian model under the controlled conditions examined. These findings suggest that the choice of inference mechanism can influence the numerical performance of fuzzy-based mathematics learning assessment, although the results are limited to the dataset, rule base, membership-function configuration, and modelling assumptions considered in this study

Keywords: Fuzzy Inference System, Mamdani inference, Sugeno inference, Gaussian membership function, Mathematics learning achievement, Educational assessment, MATLAB.

INTRODUCTION

Mathematics remains one of the most challenging subjects for secondary school students worldwide, and conventional grading systems can reduce the gradual and multidimensional nature of learning to discrete numerical classifications that may not fully reflect the nuanced judgements of experienced educators (Ertugrul, 2006). Fuzzy logic provides a framework for addressing this limitation by allowing partial membership in linguistic categories such as Poor, Average, Good, and Excellent. Consequently, fuzzy logic has been applied to student grading, performance prediction, and educational decision support (Biswas, 1995; Chen & Lee, 1999; Bai & Chen, 2008; Saleh & Kim, 2009; Voskoglou, 2013; Doz et al., 2022).

In earlier work, Abubakar et al. (2026) developed a Mamdani fuzzy inference system for assessing mathematics learning achievement using examination score, assignment completion, and class participation as input variables. They subsequently conducted a controlled comparison of Gaussian, Triangular, and Trapezoidal membership functions within the Mamdani framework. In that study, the inference mechanism, rule base, and defuzzification procedure were held constant while only the membership-function type was varied. The Gaussian model produced the strongest correlation with teacher evaluations (r = 0.9623), whereas the Triangular model produced the lowest prediction errors, with an RMSE of 7.194 and an MAE of 4.867. These findings provide a basis for examining whether the observed performance can be further influenced by another important modelling choice: the inference mechanism.

Mamdani and Sugeno systems differ primarily in the representation and processing of rule consequents. The Mamdani model uses fuzzy sets as rule consequents and requires a defuzzification procedure, such as the Centre of Gravity (COG), to obtain a crisp output (Mamdani, 1974). In contrast, the Sugeno model uses mathematical functions as rule consequents; these may be constants in a zeroth-order formulation or linear functions in a first-order formulation, with the final output obtained through weighted aggregation (Takagi & Sugeno, 1985). Mamdani systems are widely associated with linguistic interpretability and expert-knowledge representation, whereas Sugeno systems offer computational advantages and are suitable for numerical modelling and data-driven optimization (Jang et al., 1997).

Previous studies have compared Mamdani and Sugeno fuzzy inference systems in educational decision-making. Wantoro (2018), for example, compared the two approaches for selecting university students using academic and extracurricular criteria and reported that the Sugeno approach produced results closer to the reference values in that application. Similarly, Siregar and Kartika (2023) compared Mamdani and Sugeno systems for student selection using

academic grades, attendance, and academic and non-academic achievements. Although both approaches identified the same highest-ranked student, their numerical output values differed. These studies demonstrate the applicability of both inference mechanisms in educational decision-making, but their findings cannot by themselves establish whether one mechanism consistently provides better numerical agreement with an external educational benchmark.

An important methodological limitation of existing comparisons is that the two inference mechanisms are often evaluated under different combinations of datasets, input variables, membership functions, rule bases, output representations, and evaluation procedures. When several components of the modelling framework change simultaneously, differences in model performance cannot be attributed solely to the inference mechanism. Thus, a comparison designed to isolate the effect of Mamdani versus Sugeno inference requires the principal components of the fuzzy assessment system to remain unchanged while the inference mechanism is varied.

The present study addresses this methodological gap by developing a zeroth-order Sugeno variant of the previously published Mamdani-Gaussian assessment model. The same dataset of 360 secondary school students, the same three input variables, the same Gaussian membership functions, and the same 27 expert-defined IF–THEN rules are retained. The principal change is the replacement of the Mamdani fuzzy-set consequents and subsequent defuzzification procedure with constant consequents and weighted aggregation in the zeroth-order Sugeno formulation. This controlled design enables the numerical performance of the two inference mechanisms to be compared under a common educational assessment framework.

The novelty of this study therefore lies not simply in comparing Mamdani and Sugeno systems, but in attempting to isolate the effect of the inference mechanism while holding the dataset, input variables, Gaussian membership functions, and expert-defined rule base constant. Model agreement with teacher evaluations is assessed using Pearson correlation, RMSE, and MAE.

The study addresses the following research questions:

i. How does the inference mechanism (Mamdani versus Sugeno) affect the correlation between fuzzy model outputs and teacher evaluations when the membership-function type is held constant at Gaussian?

ii. How does the inference mechanism affect numerical prediction accuracy, as measured by Root Mean Square Error (RMSE) and Mean Absolute Error (MAE)?

iii. Which inference mechanism provides better numerical agreement with teacher evaluations under the controlled experimental conditions?

iv. What practical implications can be drawn regarding the selection of an inference mechanism for fuzzy-based educational assessment and decision-support systems?

MATERIALS AND METHODS

Study Design and Dataset

This study reuses the dataset and experimental context of the published Mamdani-Gaussian model (Abubakar et al., 2026): 360 secondary school student records collected from six public secondary schools in Lokoja, Kogi State, Nigeria, during the 2024–2025 academic session, using stratified random sampling (60 students per school). Three academic performance indicators served as input variables: Examination Score, Assignment Completion, and Class Participation while Learning Achievement served as the output variable, with Teacher Evaluation scores used as the external benchmark. Descriptive statistics for these variables are unchanged from the prior study and are reproduced in Table 1 for reference.

Table 1: Descriptive Statistics of the Study Variables (Reproduced from Abubakar et al., 2026)

Variable	Mean	Std. Dev.	Minimum	Maximum
Examination Score	65.053	14.347	30.000	100.000
Assignment Completion	67.731	13.668	40.000	100.000
Class Participation	62.475	14.725	30.000	100.000
Teacher Evaluation	65.086	12.565	34.000	98.667

Fuzzy Inference System Architecture

Two Mamdani and Sugeno fuzzy inference systems, differing *only* in inference mechanism, were developed under identical experimental conditions. Both models employed:

i. Three input variables: Examination Score, Assignment Completion, and Class Participation, each with three Gaussian linguistic terms (Low, Medium, High).
ii. One output variable: Learning Achievement.
iii. The same 27 expert-defined fuzzy IF–THEN rules used in the Mamdani-Gaussian model.
iv. Gaussian membership functions for fuzzification of all three inputs (Equation 3 below), with parameters unchanged from the published model.

The two architectures differ only downstream of rule evaluation:

Mamdani

fuzzy-set consequents (Very Poor, Poor, Average, Good, Excellent, each itself a Gaussian membership function on the output universe), aggregated and defuzzified via Centre of Gravity (Equations 1–2).

Sugeno (Zeroth-Order)

constant consequents z_r for each rule r, with the crisp output computed as a weighted average of rule firing strengths (Equation 5).

Mamdani Inference (Reference Model)

The firing strength of rule r is

$$\alpha_r = \min\{\mu_{A1}(x_1), \mu_{A2}(x_2), \mu_{A3}(x_3)\} \quad (1)$$

and the crisp output is obtained by Centre of Gravity defuzzification:

$$y^* = \frac{\int y\, \mu_B(y)\, dy}{\int \mu_B(y)\, dy} \quad (2)$$

Gaussian Membership Function

Both models use the same Gaussian membership function for fuzzifying inputs:

$$\mu(x; \mu_c, \sigma) = e^{-\frac{(x-\mu_c)^2}{2\sigma^2}} \quad (3)$$

where μ_c is the center and σ controls the spread of the curve. Parameters are unchanged from the published Mamdani-Gaussian model, with centers evenly distributed across each

input's range to ensure smooth transitions among neighboring linguistic categories.

Sugeno Inference (Zeroth-Order)

For the Sugeno model, each of the 27 rules takes the form

If x_1 is A_1 AND x_2 is A_2 AND x_3 is A_3 then $z_r = k$ (4)

where k_r is a constant. Rule firing strength α_r is computed identically to the Mamdani model (Equation 1, using the product or min *t*-norm). The crisp output is the firing-strength-weighted average of the rule consequents:

$$y^* = \frac{\sum_{r=1}^{27} \alpha_r \, k_r}{\sum_{r=1}^{27} \alpha_r} \quad (5)$$

To keep the comparison controlled, the constant consequent k_r for each rule was set equal to the centroid of the corresponding Mamdani output linguistic term (Very Poor, Poor, Average, Good, Excellent), so that the two models encode the same expert knowledge and differ only in how that knowledge is aggregated into a crisp score. The resulting constants were:

Table 2: Sugeno Zeroth-Order Consequent Values (Derived from Mamdani Output MF Centers)

Linguistic Term	Constant Value (k_r)
Very Poor (VP)	0.0
Poor (P)	25.0
Average (A)	50.0
Good (G)	75.0
Excellent €	100.0

Fuzzy Rule Base

The Sugeno model reuses, unmodified, the same 27 fuzzy IF–THEN rules formulated for the Mamdani model, with each rule's Mamdani output label replaced by the corresponding constant from Table 2. This ensures that any performance difference between the two models is attributable to the inference and aggregation mechanism rather than to differences in encoded expert knowledge.

MATLAB Implementation

The Sugeno fuzzy inference system was implemented using MATLAB R2024a and the Fuzzy Logic Toolbox, following the same procedure as the Mamdani model:

i. Creating a Sugeno FIS (sugfis / newfis(...,'sugeno')).
ii. Defining input variables and Gaussian membership functions identical to the Mamdani model.
iii. Constructing the 27-rule base with constant consequents (Table2).
iv. Evaluating student records using evalfis().

The generated outputs represent predicted learning achievement scores for all 360 students, directly comparable to the Mamdani-Gaussian outputs reported previously.

RESULTS AND DISCUSSION

Sugeno Fuzzy Inference System Architecture

System Architecture The generated Sugeno FIS structure is shown in Figure 1, representing the mapping from the three input evaluation variables to the single output variable via 27 constant consequents.

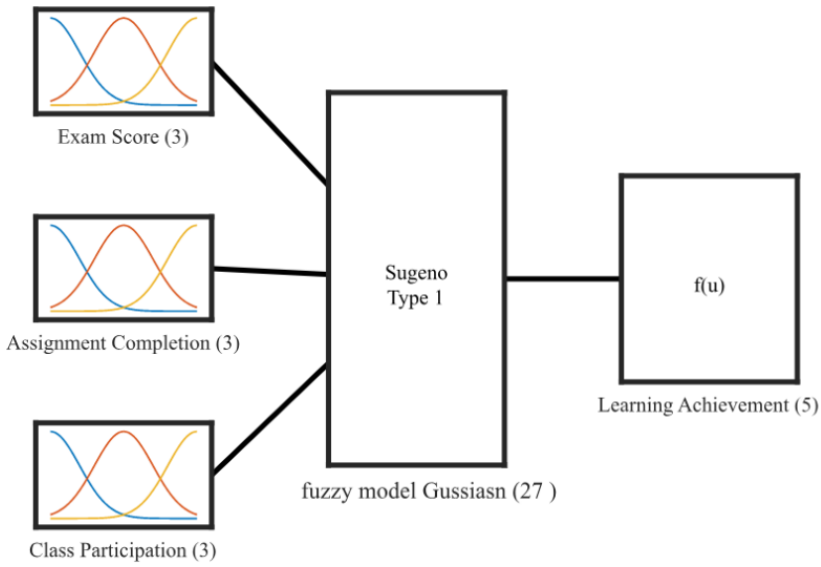


Figure 1: Sugeno Fuzzy Inference System Architecture

Input Membership Functions

Because the Sugeno model uses the same Gaussian input membership functions as the published Mamdani model, Figures 4, 7, and 10 of (Abubakar et al., 2026) (Examination Score, Assignment Completion, and Class Participation Gaussian MFs) apply unchanged and are not reproduced here.

Surface Analysis

Figure 2 displays the control surface mapping Examination Score and Assignment Completion to predicted Learning Achievement for the Sugeno model.

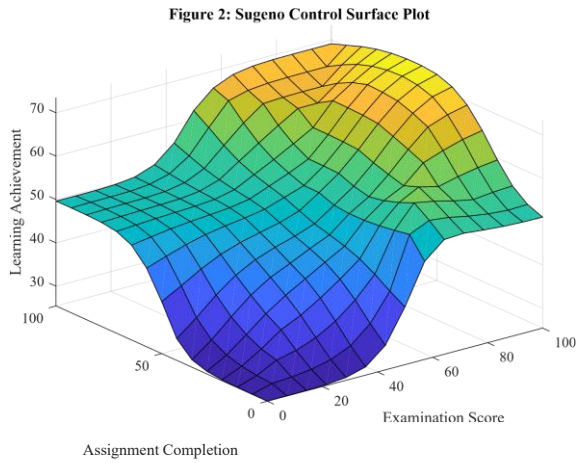


Figure 2: Sugeno Surface Plot (Learning Achievement vs. Examination Score & Assignment Completion)

The Sugeno surface displays smooth, continuous transitions across input domains, maintaining monotonic growth as input scores increase. Compared to the Mamdani surface, the zeroth order Sugeno surface eliminates minor plateau flattening near the boundaries, providing linear-like global responsiveness while retaining local smoothness from the Gaussian inputs.

Comparison with Teacher Evaluation

Figure 3 shows the student-by-student tracking performance of the Teacher Benchmark, Mamdani Gaussian model, and Sugeno-Gaussian model across all 360 records.

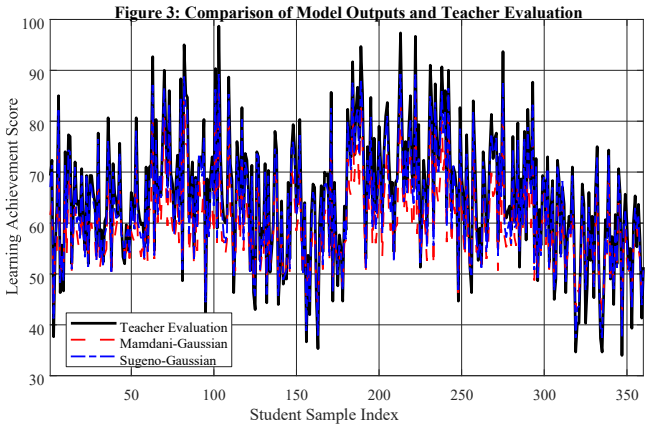
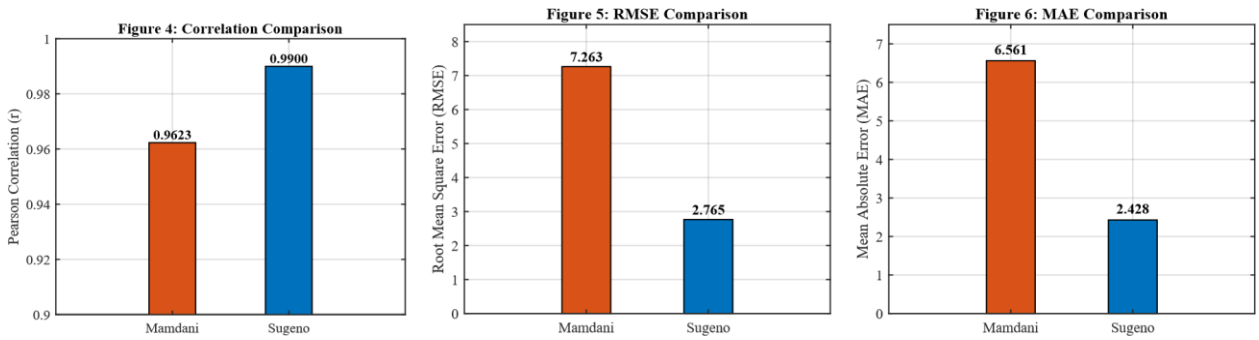


Figure 3: Comparison of Teacher Evaluation, Mamdani-Gaussian, and Sugeno-Gaussian Outputs Across Student Samples

Quantitative Performance Evaluation

Table 3: Performance Comparison of Mamdani-Gaussian and Sugeno-Gaussian Models

Model	Correlation	RMSE	MAE
Mamdani-Gaussian (Abubakar et al., 2026)	0.9623	7.263	6.561
Sugeno-Gaussian (this study)	0.98996	2.7651	2.4279



(a) Correlation (*r*) (b) RMSE (c) MAE

Figure 4: Performance Metrics Comparison between Mamdani-Gaussian and Sugeno-Gaussian Models

Discussion

The results indicate that, under the experimental configuration considered in this study, the zeroth-order Sugeno-Gaussian model produced closer agreement with the teacher evaluation scores than the Mamdani-Gaussian model. The Sugeno-Gaussian model achieved a Pearson correlation coefficient of r=0.98996r=0.98996, compared with r=0.9623r=0.9623 for the Mamdani-Gaussian model. In addition, the Sugeno-Gaussian model recorded an RMSE of 2.7651 and an MAE of 2.4279, compared with 7.2630 and 6.5607, respectively, for the Mamdani-Gaussian model. Thus, relative to the Mamdani-Gaussian model, the Sugeno-Gaussian model reduced RMSE by approximately 61.9% and MAE by approximately 63.0%. These differences indicate substantially smaller prediction errors for the Sugeno-Gaussian model on the 360 student records considered in this study.

The difference between the two models can be considered in relation to their output representations. In the Mamdani system, rule consequents are fuzzy output sets that are aggregated and subsequently converted to a crisp value using Centre of Gravity defuzzification. In the zeroth-order Sugeno system, the rule consequents are constant values and the final output is obtained from the firing-strength-weighted average of these constants. Consequently, the two systems use different mechanisms for transforming the activated rules into a numerical achievement score. The observed performance difference is therefore consistent with the different output aggregation procedures. However, because the Sugeno consequent constants were derived from the corresponding Mamdani output terms, and because the Mamdani results used for comparison originated from the previously developed model, the observed improvement should not be interpreted as establishing that the aggregation mechanism alone caused the difference.

The choice of consequent representation may also contribute to the observed numerical behavior. The Sugeno model directly combines the constant consequents according to their rule firing strengths, whereas the Mamdani model first constructs an aggregated output fuzzy set and then obtains a crisp value through defuzzification. The smoother response observed in the Sugeno surface is consistent with the weighted-average formulation of the zeroth-order model. Nevertheless, this interpretation is descriptive rather than causal, since additional controlled experiments would be required to determine the separate contribution of the consequent values, aggregation procedure, and defuzzification method.

The Sugeno control surface also shows a smooth variation in predicted learning achievement across the examined input space. The output generally increases as examination score and assignment completion increase, which is consistent with the intended ordering of the linguistic categories and consequent values. This behavior provides qualitative evidence of the expected monotonic response in the displayed surface. A formal monotonicity test, however, was not performed; therefore, the surface should be interpreted as demonstrating the observed response pattern rather than establishing global mathematical monotonicity over the entire input domain.

The results are consistent with the general distinction between Mamdani and Sugeno fuzzy inference systems reported in the fuzzy-systems literature. Mamdani systems provide linguistic fuzzy-set consequents and are particularly suitable when interpretability and explicit expert knowledge are important, whereas Sugeno systems produce numerical outputs efficiently through functional or constant consequents. In the present application, the numerical performance of the zeroth-order Sugeno-Gaussian model was better according to all three evaluation measures considered. This finding suggests that the Sugeno formulation may be advantageous for this particular educational assessment configuration when prediction accuracy relative to teacher evaluation is the primary criterion.

The comparison should nevertheless be interpreted within the scope of the present experimental setting. The dataset contains 360 secondary school students from six public secondary schools in Lokoja, Kogi State, and the models use examination score, assignment completion, and class participation as inputs. Both systems employ Gaussian input membership functions and the same expert-defined 27-rule structure, while the Sugeno model uses constant consequents corresponding to the five output linguistic categories. Therefore, the observed advantage should be regarded as evidence for the performance of the specified zeroth-order Sugeno-Gaussian configuration on this dataset, rather than as evidence that Sugeno inference is universally superior to Mamdani inference in educational assessment.

Limitations

This study is subject to several limitations. First, the data were obtained from a single geographical location and one educational level; consequently, the results may not generalize directly to other regions, school systems, or age groups. Second, the comparison considered a zeroth-order Sugeno system with constant consequents and did not investigate first-order Sugeno models with linear consequents or ANFIS-based parameter optimization. Third, the Mamdani-Gaussian results used for comparison were obtained from the previously developed Mamdani model. Although the present study retained the same input variables, Gaussian membership functions, and expert-defined rule structure, differences associated with implementation and computational settings should be considered when interpreting the comparison. Future studies should rerun both architectures within the same computational pipeline and evaluate paired prediction errors, residual patterns, and uncertainty estimates. Such analyses would provide a stronger basis for determining the robustness of the observed performance difference.

Conclusion

This study compared the performance of Mamdani and zeroth-order Sugeno fuzzy inference systems using Gaussian membership functions for evaluating mathematics learning achievement. The comparison was conducted using the same three input variables, the same expert-defined 27-rule structure, and the same educational assessment framework for 360 secondary school students. The results showed that, for the dataset and configuration examined, the Sugeno-Gaussian model achieved a higher Pearson correlation with teacher evaluations (r=0.98996r=0.98996) and lower prediction errors (RMSE =2.7651=2.7651; MAE =2.4279=2.4279) than the Mamdani-Gaussian model (r=0.9623r=0.9623; RMSE =7.2630=7.2630; MAE =6.5607=6.5607).

These findings indicate that the zeroth-order Sugeno-Gaussian configuration provided better numerical agreement with the teacher evaluation scores than the corresponding Mamdani-Gaussian configuration in the present study. The result should, however, be interpreted as an empirical finding specific to the dataset, membership-function configuration, rule base, and consequent values examined, rather than as evidence of universal superiority of Sugeno inference over Mamdani inference.

The study demonstrates the potential of fuzzy inference systems for representing the gradual and multidimensional nature of mathematics learning achievement. Future research should examine first-order Sugeno systems with linear consequents, ANFIS-based parameter optimization, larger and geographically diverse datasets, and additional statistical or resampling-based analyses of prediction errors. Such extensions would help determine the robustness and generalizability of the observed performance differences between the two inference mechanisms.

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