

An α -Cut-Based Ranking Index for Normal Trapezoidal Fuzzy Numbers

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		ABSTRACT
Received:	19 Oct. 2026	This paper proposes a simplified ranking approach for normal trapezoidal fuzzy numbers using an integrated $\alpha - cut$ ranking index (Γ^{α}). The fundamental arithmetic operations for $\alpha - cuts$ are reviewed to compute the proposed index. The ranking performance is evaluated across seven sets of fuzzy numbers, comparing the results against nine established existing methods. Comparative analysis demonstrates that the proposed approach is straightforward to apply, effectively better, and yields high consistency with existing literature while significantly reducing computational complexity.
Accepted:	26 Oct. 2026	
Published:	28 Oct. 2026	
Keywords:		Fuzzy Set, Fuzzy Number, Normal Trapezoidal Fuzzy Number, A-Cut, Ranking Index

INTRODUCTION

In 1965, Zadeh introduced the concept of fuzzy set theory to tackle the real-world decision-making problems that are persistent to uncertain (or) vague in a number of ways. Since then, the fuzziness of a fuzzy set can be represented in number of ways, as extended by different researchers. One of the most powerful and useful representation is by membership function. Fuzzy number is an extension of fuzzy set. It can also be classified in different forms, depending on the nature of the fuzzy number, such as Triangular Fuzzy Numbers (TFNs), Trapezoidal Fuzzy Numbers, Interval Fuzzy Numbers etc.

Fuzzy sets and fuzzy numbers have been extensively applied in decision-making, optimization, engineering, economics, risk analysis and artificial intelligence. A fuzzy number is particularly useful when a numerical quantity cannot be specified precisely but can instead be described by a range of possible values with different membership degrees. Among the various listed representations of fuzzy numbers, trapezoidal fuzzy numbers are special fuzzy numbers, because of their simple structure of quadruple $\tilde{A} = (a, b, c, d)$, where a and d represent the lower and upper boundaries of the support, while b and c determine the core. An α -cut is one of the most important analytical aspects of fuzzy numbers. For a normal trapezoidal fuzzy number, its α -cut is an interval whose endpoints vary linearly with α . This representation of α -cut is standard in fuzzy arithmetic and fuzzy-number analysis. This is because of its significant responsibilities for applications and generalizations. The concept was discussed in numerous contributions including; (Kaufmann & Gupta, 1988), (Amiya & Madhumangal, 2007), (Kumar *et al.*, 2010), etc.

The method of ranking fuzzy numbers was first proposed by (Jain, 1976) for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. (Yager, 1981) proposed three indices for the purpose of ordering fuzzy quantities. (Bortolan & Degani, 1985) reviewed some of the methods (Jain, 1976) & (Yager, 1981) for ranking fuzzy subsets. (Kaufmann & Gupta, 1988) proposed a simplified approach for the ranking of fuzzy numbers. (Campos and Gonzalez, 1985) proposed a subjective approach for ranking fuzzy numbers. (Liou & Wang, 1992) developed a ranking method based on integral value index. (Cheng, 1998) presented a method for ranking fuzzy numbers using the distance method. (Kwang & Lee, 1999) considered

possibility distribution characteristic of fuzzy numbers in their ranking. (Modarres & Nezhad, 2001) proposed a ranking method based on preference function which measures fuzzy numbers point by point and at each point the most preferred number is identified. (Chu & Tsao, 2002) proposed a method of ranking fuzzy numbers based on the area between the centroid point and original point. (Deng & Liu, 2004) presented a centroid-index method for ranking fuzzy numbers. (Chen & Chen, 2007) presented a method of ranking of generalized trapezoidal fuzzy numbers. (Wang & Lee, 2009) used the concept of centroid in developing a ranking index. (Chen & Tang, 2008) proposed a method for ranking p-norm trapezoidal fuzzy numbers.

Abbasbandy & Hajjari (2009) introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α -levels of trapezoidal fuzzy numbers. Chen & Chen, (2007) presented a method for fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. (Kumar *et al.*, 2011) proposed a method for ranking generalized trapezoidal fuzzy numbers based on rank, mode, divergence and spread. (Rezvani, 2012) proposed a new method for ranking in perimeters of two generalized trapezoidal fuzzy numbers. (Babu *et al.*, 2012) proposed a method of ranking generalized trapezoidal fuzzy numbers using centroid of centroids. (Singh *et al.*, 2016) presented a method of ranking generalized trapezoidal fuzzy numbers by section wise perimeter with application to risk analysis. (Datti *et al.*, 2017) proposed a method of Ranking triangular fuzzy numbers using the area between the original point and the incentre.

The importance of α -cuts becomes particularly clear in fuzzy-number ranking. Numerous ranking procedures have been proposed, including maximizing/minimizing-set approaches, centroid-based methods, distance methods, spread-based methods and some approaches are based explicitly on selected α -levels, for instance, (Abbasbandy and Hajjari, 2009) utilizes the concepts presented in (Liou & Wang, 1992) and developed a ranking approach for trapezoidal fuzzy numbers based on left and right spreads at selected α -levels. Some other approaches are; (Wang *et al.*, 2009), (Basirzadeh & Abbasi, 2008), (Abbasbandy & Asady, 2006), (Chu & Tsao, 2002), etc.

The progressive development of ranking methods indicates that no single universally accepted ranking procedure exists. This observation motivates us to propose a method of ranking

that can consider the α – cut of a normal trapezoidal fuzzy number as the ranking index. The proposed work is a modified version of midpoint approach presented in (Liou & Wang, 1992), and adopted and extended by many researches as shown above in different literatures. The method should be evaluated with the existing powerful methods of ranking fuzzy numbers, and a comparative table of the results would be provided.

The main objective of this paper is therefore to propose a method of ranking of a normal trapezoidal fuzzy number by its α – cut, a modified midpoint of a normal trapezoidal fuzzy number is considered as its α – cut ranking index, and then to compute a ranking approach that can compete with some existing ranking approaches in the literature.

There are numerous existing methods of ranking fuzzy numbers by indices that considers different approaches such as (Liou & Wang, 1992), (Chu & Tsao, 2002), etc. The methods acted on different directions based on their applicability and the choice of index. This approach provides a direct, computationally streamlined closed-form expression for normal trapezoidal fuzzy numbers without requiring complex numerical integration steps.

MATERIALS AND METHODS

Preliminaries

Definition 2.1: Fuzzy Set (Zadeh, 1965)

The characteristic function $\mu_{\tilde{A}}$ of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X (Zadeh, 1965). This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set $\tilde{A}$. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{x, \mu_{\tilde{A}}(x); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set. More detailed discussion can be found in (Zimmermann, 1996).

Definition 2.2: Normal Fuzzy Set

A fuzzy subset $\tilde{A}$ of universal set U is normal iff $\sup_{x \in U} \mu_{\tilde{A}}(x) = 1$, where U is the universe set.

Definition 2.3: Fuzzy Number

Fuzzy set $\tilde{A}$ defined on the universal set of a real number R is said to be a fuzzy number if its membership function has the following characteristics:

- (i) $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is continuous.
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
- (iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
- (iv) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b, c]$, where $a \leq b \leq c \leq d$.

Definition 2.4: A Trapezoidal Fuzzy Number (Asady, 2010)

A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a \leq x \leq b \\ 1, & b \leq x \leq c \\ \frac{(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

Since both L and R of trapezoidal fuzzy number are linear, A trapezoidal fuzzy number can be represented by quadruple $(a, b, c, d) \in R^4$ and is completely characterized by $a \leq b \leq c \leq d$. In this case the r-level sets are given by $\underline{u}^r = a + r(b-a)$ and $\bar{u}^r = d + r(d-c)$. If we have $b = c$ the quadruple reduces to triple (a, b, c) which is a triangular fuzzy number.

Definition 2.5: A Generalized Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the universal set of real number R is said to be a generalized fuzzy number if its membership function has the following characteristics.

- (i) $\mu_{\tilde{A}} : \mathbb{R} \rightarrow [0,1]$ is continuous,
- (ii) $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$,
- (iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$,
- (iv) $\mu_{\tilde{A}}(x) = w$ for all $x \in [b, c]$, where $0 < w \leq 1, \forall a, b, c, d \in \mathbb{R}$.

Definition 2.6: A Generalized Trapezoidal Fuzzy Number (Rezvani, 2012)

A normal trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by: $\mu_{\tilde{A}}(x) =$

$$\begin{cases} \frac{w(x-a)}{(b-a)}, & a \leq x \leq b \\ w, & b \leq x \leq c \\ \frac{w(x-d)}{(c-d)}, & c \leq x \leq d \end{cases}$$

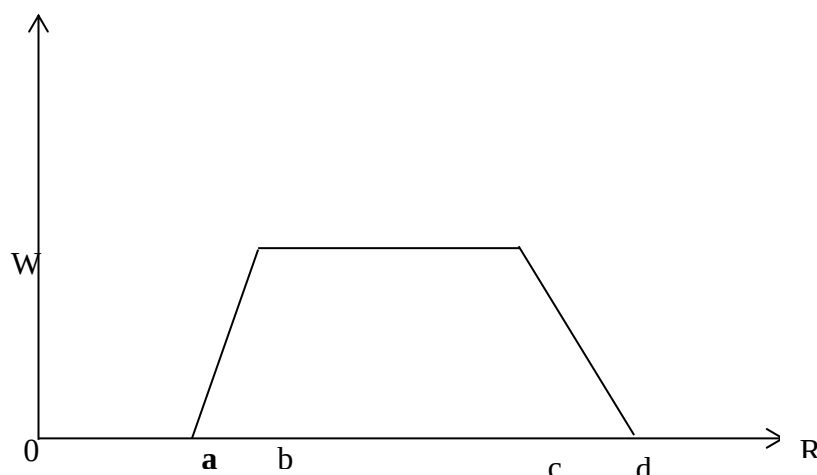


Figure 1: A Generalized Trapezoidal Fuzzy Number

Definition 2.7: Level set: An α – level set of a Fuzzy Number $\tilde{A}$ is a Non- Fuzzy Set Denoted by $\tilde{A}_\alpha$ and is Defined by

$$\tilde{A}_\alpha = \{x \mid \mu_{\tilde{A}}(x) \geq \alpha\}, \quad 0 < \alpha \leq 1$$

Remark 2.1: When α -level set is an empty set $\emptyset$, the following holds.

- i. $\tilde{A}_\alpha + \Phi = \Phi$ and
- ii. $\tilde{A}_\alpha \times \Phi = \Phi$.

Definition 2.8: Height

Suppose $\tilde{A}: X \rightarrow [0,1]$ is a fuzzy set, then the height of $\tilde{A}$ is denoted by $h(\tilde{A})$ and is defined as $h(\tilde{A}) = \vee \tilde{A}(x) = \sup\{\mu_{\tilde{A}}(x) : x \in X\}$. This is the highest membership grade obtained by any element in that set. (Klir & Yuan, 1995).

Definition 2.9: Support

Suppose $\tilde{A}: X \rightarrow [0,1]$ is a fuzzy set, then the “Support of” is the ordinary subset of X , that is the support of fuzzy set $\tilde{A}$ within a universal set X is the crisp set that contains all the elements of X that have non-zero membership grades in $\tilde{A}$. Symbolically, $\text{sup } \tilde{A} = \{x \in X, \mu_{\tilde{A}}(x) > 0\}$. (Klir & Yuan, 1995).

Definition 2.7: Core

Suppose $\tilde{A}: X \rightarrow [0,1]$ is a fuzzy set, then “Core of $\tilde{A}$ ” is denoted by $\text{Core}(\tilde{A})$ and defined as follows: $\text{Core}(\tilde{A}) = \{x \in X, \mu_{\tilde{A}}(x) = 1\}$. (Klir & Yuan, 1995).

α – cut of a Trapezoidal Fuzzy Number

The α – cut of a normal trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, is defined by:

$$[\tilde{A}]_\alpha = \{x \in \mathbb{R} : \mu_{\tilde{A}}(x) \geq \alpha\} \quad (1)$$

For every $\alpha \in [0, 1]$, $[\tilde{A}]_\alpha = [a + \alpha(b - a), d - \alpha(d - c)]$

For the α – level, the 0- level set of a normal trapezoidal fuzzy number can be express as;

$[\tilde{A}]_0 = \{x : \mu_{\tilde{A}}(x) > 0\}$, thus $[\tilde{A}]_0 = [a, d]$ and $[\tilde{A}]_1 = [b, c]$ are respectively the support and the core of a normal trapezoidal fuzzy number.

The α – cut of a normal trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$, can be represented as lower and upper intervals as presented respectively:

$$L_{\tilde{A}}(\alpha) = a + \alpha(b - a), \text{ and } R_{\tilde{A}}(\alpha) = d - \alpha(d - c) \quad (2)$$

$$\therefore [\tilde{A}]_\alpha = [L_{\tilde{A}}(\alpha), R_{\tilde{A}}(\alpha)].$$

Operation in Fuzzy Number by α -Level. (Zimmermann, 1966)

Let $\tilde{A}_\alpha$ and $\tilde{B}_\alpha$ be α -level sets of a convex fuzzy numbers $\tilde{A}$ and $\tilde{B}$ respectively, then the α -level sets are intervals in $\mathbb{R}$, which are special convex fuzzy numbers whose grades are unity at X belonging to $\tilde{A}_\alpha$ and zero elsewhere. Let the α -level sets, say, $\text{sum } \tilde{A} + \tilde{B}$ of $\tilde{A}$ and $\tilde{B}$ be denoted by $(\tilde{A} + \tilde{B})_\alpha$, we then obtain $(\tilde{A} + \tilde{B})_\alpha = \tilde{A}_\alpha + \tilde{B}_\alpha$. In other words, the α -level set $(\tilde{A} + \tilde{B})_\alpha$ is the sum of α -level sets $\tilde{A}_\alpha$ and $\tilde{B}_\alpha$. The general operation can be performed as:

$\tilde{A} + \tilde{B} = \int_0^1 \alpha (\tilde{A} + \tilde{B})_\alpha = \int_0^1 \alpha (\tilde{A}_\alpha + \tilde{B}_\alpha)$, similarly, we can obtain:

$$\tilde{A} - \tilde{B} = \int_0^1 \alpha (\tilde{A} - \tilde{B})_\alpha = \int_0^1 \alpha (\tilde{A}_\alpha - \tilde{B}_\alpha)$$

$$\tilde{A} \times \tilde{B} = \int_0^1 \alpha (\tilde{A} \times \tilde{B})_\alpha = \int_0^1 \alpha (\tilde{A}_\alpha \times \tilde{B}_\alpha)$$

$$\tilde{A} \div \tilde{B} = \int_0^1 \alpha (\tilde{A} \div \tilde{B})_\alpha = \int_0^1 \alpha (\tilde{A}_\alpha \div \tilde{B}_\alpha)$$

The defined operation can be extended to normal trapezoidal fuzzy number as:

$$(\tilde{A} + \tilde{B})_\alpha = [a_1 + a_2 + \alpha((b_1 - a_1) + (b_2 - a_2)), d_1 + d_2 - \alpha((d_1 - c_1) + (d_2 - c_2))] \quad (3)$$

Also,

$$(k\tilde{A})_\alpha = [k(a + \alpha(b - a)), k(d - \alpha(d - c))] \quad (4)$$

Ranking Generalized Trapezoidal Fuzzy Number Using its α – cut Ranking Index

Let $\tilde{A} = (a, b, c, d)$ be a normal trapezoidal fuzzy number. The α – mean index which is the midpoint of the α – cut intervals, is denoted by $M_\alpha(\tilde{A})$, and can be presented as:

$$M_\alpha(\tilde{A}) = \frac{L_{\tilde{A}}(\alpha) + R_{\tilde{A}}(\alpha)}{2} = \frac{(a + \alpha(b - a)) + (d - \alpha(d - c))}{2} = \frac{1}{2} [(a + \alpha(b - a)) + (d - \alpha(d - c))] \quad (5)$$

Integrating $M_\alpha(\tilde{A})$ for $\alpha \in [0, 1]$, we have the ranking index as:

$$\Gamma^\alpha = \int_0^1 M_\alpha(\tilde{A}) d\alpha = \frac{a+d}{2} + \frac{(b-a)-(d-c)}{4} \quad (6)$$

Theorem 3.1: Let $\tilde{A} = (a_1, b_1, c_1, d_1; 1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; 1)$ be two normal trapezoidal fuzzy numbers. Where $a_1 \leq b_1 \leq c_1 \leq d_1 \in \mathbb{R}$ and $a_2 \leq b_2 \leq c_2 \leq d_2 \in \mathbb{R}$, and $\Gamma^\alpha(\tilde{A})$ and $\Gamma^\alpha(\tilde{B})$ are the α – cut ranking indexes of the two normal trapezoidal fuzzy numbers $\tilde{A}$ and $\tilde{B}$. Then:

- (i) If $\Gamma^\alpha(\tilde{A}) < \Gamma^\alpha(\tilde{B})$, then $\tilde{A} < \tilde{B}$,
- (ii) If $\Gamma^\alpha(\tilde{A}) > \Gamma^\alpha(\tilde{B})$, then $\tilde{A} > \tilde{B}$,
- (iii) If $\Gamma^\alpha(\tilde{A}) \sim \Gamma^\alpha(\tilde{B})$, then $\tilde{A} \sim \tilde{B}$.

RESULTS AND DISCUSSION

In this subsection, similar to other approaches, seven sets of fuzzy numbers are considered for comparing the proposed approach with other existing approaches. The results are presented in a tabular form in the end.

Example 1. Seven sets of numbers are used and the results are compared with many existing approaches in table (1).

Set 1: Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 1)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.5 \text{ and } \Gamma^\alpha(\tilde{B}) = 0.25$$

Since $\Gamma^\alpha(\tilde{A}) > \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} > \tilde{B}$ holds.

Set 2: Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.3 \text{ and } \Gamma^\alpha(\tilde{B}) = 0.3$$

Since $\Gamma^\alpha(\tilde{A}) \approx \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} \sim \tilde{B}$ holds.

Set 3: Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (1, 1, 1, 1; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.3 \text{ and } \Gamma^\alpha(\tilde{B}) = 1$$

Since $\Gamma^\alpha(\tilde{A}) < \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} < \tilde{B}$ holds.

Set 4: Let $\tilde{A} = (-0.5, -0.3, -0.3, -0.1; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$.

$$\Gamma^\alpha(\tilde{A}) = -0.3 \text{ and } \Gamma^\alpha(\tilde{B}) = 0.3$$

Since $\Gamma^\alpha(\tilde{A}) < \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} < \tilde{B}$ holds.

Set 5: Let $\tilde{A} = (0.3, 0.5, 0.5, 1; 1)$ and $\tilde{B} = (0.1, 0.6, 0.6, 0.8; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.575 \text{ and } \Gamma^\alpha(\tilde{B}) = 0.525$$

Since $\Gamma^\alpha(\tilde{A}) > \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} > \tilde{B}$ holds.

Set 6: Let $\tilde{A} = (0.0, 0.4, 0.6, 0.8; 1)$, $\tilde{B} = (0.2, 0.5, 0.5, 0.9; 1)$ and $\tilde{C} = (0.1, 0.6, 0.7, 0.8; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.45, \Gamma^\alpha(\tilde{B}) = 0.525 \text{ and } \Gamma^\alpha(\tilde{C}) = 0.55$$

Since $\Gamma^\alpha(\tilde{A}) < \Gamma^\alpha(\tilde{B}) < \Gamma^\alpha(\tilde{C})$, $\Rightarrow \tilde{A} < \tilde{B} < \tilde{C}$ holds.

Set 7: Let $\tilde{A} = (0.1, 0.2, 0.4, 0.5; 1)$ and $\tilde{B} = (-2, 0, 2; 1)$.

$$\Gamma^\alpha(\tilde{A}) = 0.3, \Gamma^\alpha(\tilde{B}) = 0$$

Since $\Gamma^\alpha(\tilde{A}) > \Gamma^\alpha(\tilde{B})$, $\Rightarrow \tilde{A} > \tilde{B}$ holds.

Table 1: Comparison of The Ranking Results for Different Approaches

Approaches	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7
(Cheng, 1998)	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	Error	$\tilde{A} \sim \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	Error
(Chu & Tsao, 2002)	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	Error	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	Error
(Chen & Chen, 2007)	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
(Abbasbandy & Hajjari, 2009)	Error	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
(Chen & Chen, 2009)	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
(Kumar <i>et al.</i> , 2011)	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
(Babu <i>et al.</i> , 2012)	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} = \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$
(Rezvani, 2012)	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} > \tilde{B} > \tilde{C}$	$\tilde{A} < \tilde{B}$
(Singh <i>et al.</i> , 2012)	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} \sim \tilde{B}$	$\tilde{A} > \tilde{C} > \tilde{B}$	$\tilde{A} < \tilde{B}$
Proposed Approach	$\tilde{A} > \tilde{B}$	$\tilde{A} \approx \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} < \tilde{B}$	$\tilde{A} > \tilde{B}$	$\tilde{A} < \tilde{B} < \tilde{C}$	$\tilde{A} > \tilde{B}$

Discussion on the Comparison Table

From the table above, the results of the proposed approach for ranking have coincides with the following existing methods of ranking fuzzy numbers;

- Set 1:** (Kumar *et al.*, 2011)
- Set 2:** (Abbasbandy & Hajjari, 2009), (Chu & Tsao, 2002), (Cheng, 1998) and (Kumar *et al.*, 2011)
- Set 3:** (Chen & Chen, 2007), (Abbasbandy & Hajjari, 2009), (Babu *et al.*, 2012) (Kumar *et al.*, 2011), and (Chen & Chen, 2009).
- Set 4:** (Chu & Tsao, 2002), (Chen & Chen, 2007), (Babu *et al.*, 2012) (Kumar *et al.*, 2011), and (Chen & Chen, 2009).
- Set 5:** (Chu & Tsao, 2002), (Cheng, 1998), (Chen & Chen, 2007), (Kumar *et al.*, 2011), and (Chen & Chen, 2009).
- Set 6:** Coincides with all the methods except (Babu *et al.*, 2012), (Rezvani, 2012) and (Singh *et al.*, 2012).

- Set 7:** Coincides with all the methods except (Rezvani, 2012), (Singh *et al.*, 2012) and the methods of (Cheng, 1998) and (Chu & Tsao, 2002) that could not be able to rank the fuzzy numbers.

Remark 3.1: The results of the proposed approach coincide completely with the results of (Kumar, *et al.*, 2011), but the proposed approach is easier in terms of computation and general application than (Kumar, *et al.*, 2011) and all the existing approaches presented above.

General Comparison of the Ranking Index on a Bar Graph

The ranking index (Γ^{α}) of all the seven sets of normal trapezoidal fuzzy numbers have been compared via Bar graph as shown below:

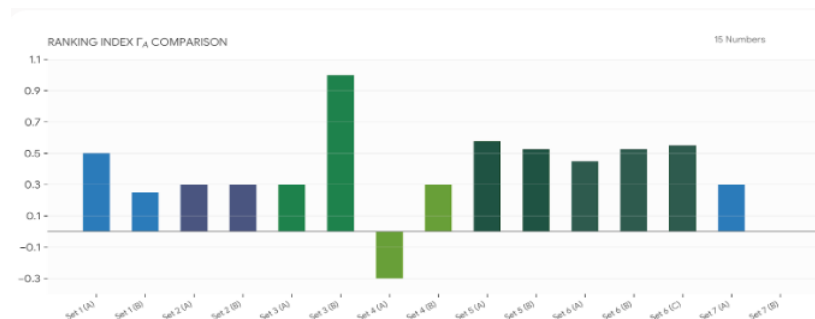


Figure 2: Representation of The Ranking Index On a Bar Graph

General Comparison of the Fuzzy Numbers Base on their Membership Grade

The complete sets of the sampled fuzzy numbers have been

presented and compared based on their membership grades as shown below:

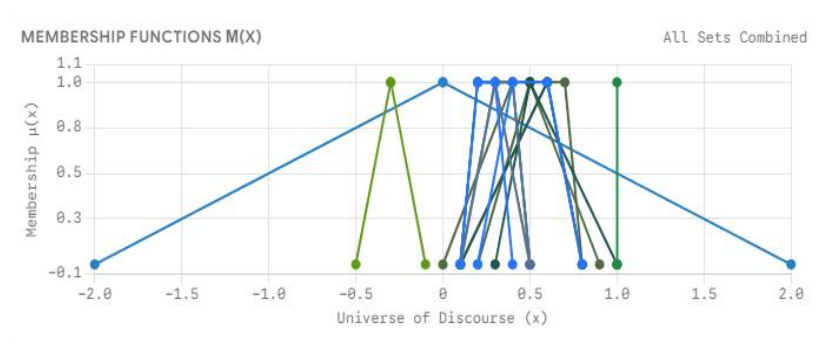


Figure 3: Representation of Fuzzy Numbers by Membership Grade

Effectiveness of the Proposed Ranking Index

Based on the ranking index bar chart and the membership function plots of the seven (7) sets of normal trapezoidal fuzzy numbers, the conclusion and analysis can be summarized through the following key insights:

1. Based on monotonicity and consistency of the proposed α – cut ranking index (Γ^α), fuzzy numbers are expected to be ranked higher if they are positioned further to the right on x-axis [e.g. observed that the fuzzy number $\tilde{B} = (1,1,1,1; 1)$ in set 3 has the overall ranking index value $\Gamma^\alpha = 1$].
2. The proposed method can effectively differentiate and rank non-overlapping and partially overlapping fuzzy numbers (e.g. $\tilde{A} < \tilde{B}$ in set 1 and $\tilde{A} < \tilde{B} < \tilde{C}$ in set 6).
3. The proposed method can effectively rank two or more fuzzy numbers with equivalence symmetric span [e.g. in set 2, $\tilde{A} = (0.1,0.2,0.4,0.5; 1)$ and $\tilde{B} = (0.1,0.3,0.3,0.5; 1)$, they shared identical bound $[0.1, 0.5]$ resulting to the same ranking index $\Gamma^\alpha(\tilde{A}) = \Gamma^\alpha(\tilde{B}) = 0.3$].
4. The proposed method can handle and rank fuzzy numbers in negative domains smoothly (e.g., as seen in set 4 and set 7).

CONCLUSION

This study demonstrates that the proposed α – cut ranking index (Γ^α) provides a computationally streamlined closed-form approach for normal trapezoidal fuzzy numbers, yielding results that align closely with established literature across overlapping intervals, symmetric spans with tied ranks (e.g., Set 2), and negative domains. However, because these verified findings are derived from seven selected sets of normal trapezoidal fuzzy numbers, further empirical validation on broader, larger-scale datasets and complex decision-making applications is recommended to fully establish the effectiveness and operational boundaries of this method.

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