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Mathematical and Scenario Analysis of an Improved Six-Compartment Japa Model Incorporating Anticipated Rewards and Talent Retention Using LADM and NBS 2023 Population Data

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ABSTRACT
In this work, we develop and analyze a six-compartmental mathematical model on brain drain dynamics within a source country. The compartments of interest include the source population, employment opportunities, frustrated population due to corruption, unemployment population, immigrants, and emigrants. In addition, the work explores the impacts of anticipated rewards, talent retention, and media awareness on changes in the above compartmental populations. We explore non-negativity and boundedness of the system, determine brain-drain-free and persistent equilibria, compute the basic reproduction number, and investigate parameter sensitivity. For the initial population, we use statistical data of Nigeria's National Bureau of Statistics while others are from the literature or are assumed. With the updated baseline parameter set, the value of R_0 is 0.0126874 which is less than one and meets the threshold condition for the local asymptotic stability of the brain-drain-free equilibrium under the assumed conditions. Numerical solution of the model is obtained using the restarted two-correction scheme of Laplace-Adomian decomposition method (LADM) algorithm and analyzed using Radau IIA and LSODA algorithms. With reduction of the time step, the maximum scaled error is decreased from 8.20×10^{-6} to $8.19x10^{-8}$while the maximum scaled difference between Radau IIA and LSODA is approximately $1.82x10^{-11}$.The Scenario results provide away to compare how selected parameter values relate to the modeled corruption-frustrated and unemployment trajectories.

Keywords: Brain Drain, Talent Retention, Anticipated Rewards, Media Awareness, LADM, Sensitivity Analysis

INTRODUCTION

Brain drain refers to the emigration of skilled individuals from their country of origin resulting in the reduced availability of human capital in crucial sectors to support economic and institutional development (Docquier & Rapoport, 2007). The term Japa, common in Nigeria, refers to emigration or the act of leaving one's country in search of better employment and living conditions overseas (Adeniji et al.,2024).

Migration decisions depend not only on present economic conditions but also on potential future opportunities, individual aspirations, access to information, and other tangible and intangible aspects influencing the timing, intention, and location of migration (Czaika et al., 2021).

(Adeniji et al.,2024) introduced a mathematical "Japa model" representing the source population, employment opportunities, emigrants, and returnees. The current study builds upon this by introducing two (2) state variables; unemployed individuals and the population of corrupt individuals and incorporates anticipated rewards, talent retention, and media awareness factors that could influence the rates of migration. This addition enables the analysis of labour-market and corruption-related issues and their implications for emigration and return migration within the confines of the proposed system (Adeniji et al.,2024). The total projected population for 2022 (216,783,381) from (NBS, 2023) was used as the baseline for the source population (NBS,2023). The six variables source population, available employment opportunities, population of corrupt individuals, unemployed people, emigrants), and returnees play different roles in the system. Available employment opportunities have been incorporated to reflect the home labour market's role in the migration processes described by the model. Similarly, the study includes unemployment and corruption as control variables since the research focuses on how these factors influence emigration and return migration. Finally, anticipated rewards, talent retention, and media awareness are considered important model factors; thus, their relationships with the other variables should be interpreted with caution as the indicated equations and scenario analyses assume specific underlying theoretical expectations rather than presenting these associations as guaranteed policy outcomes. Return migration presents additional importance as returning migrants could contribute to the overall skill-endowed population and affect broader re-entry conditions depending on their competencies and the environment upon return (King & Kuschminder,2022).

Therefore, this study aims to formulate and analyse a six-compartment extension of the Japa model that incorporates employment opportunities, corruption, unemployment, emigration, and return migration and investigates the assumed roles of anticipated rewards, talent retention, and media awareness using equilibrium, threshold, sensitivity, and numerical-scenario analyses.

Mathematical Model Formulation

The model equation is;

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta(1-\varepsilon)SD - \alpha SP - \mu S + \gamma R + \rho R + \eta SC + \sigma SC \\
\frac{dP}{dt} &= r_0\left(1-\frac{P}{K}\right)P - r_1 P - \varphi CP - \psi PU - \mu P \\
\frac{dC}{dt} &= \varphi CP - \eta SC - \sigma SC - \mu C \\
\frac{dU}{dt} &= \psi PU - \pi UR - \mu U \\
\frac{dD}{dt} &= \beta(1-\varepsilon)SD - \theta DP - xDe^{-MD} - \mu D \\
\frac{dR}{dt} &= \theta DP + \alpha SP + xDe^{-MD} - \gamma R - \rho R - \mu R + \pi UR
\end{aligned} \qquad (1)$$

Table 1: Nigeria Total Projected Population Distribution by Sex, 2018–2022

Year	Total	Male	Female
2018	196,042,933	98,263,945	97,778,988
2019	201,135,262	100,739,423	100,395,839
2020	206,283,338	103,242,979	103,040,359
2021	211,493,324	105,776,862	105,716,462
2022	216,783,381	108,350,410	108,432,971

Source: National Bureau of Statistics (2023). Annual Abstract of Statistics 2023, Table 13, p. 16

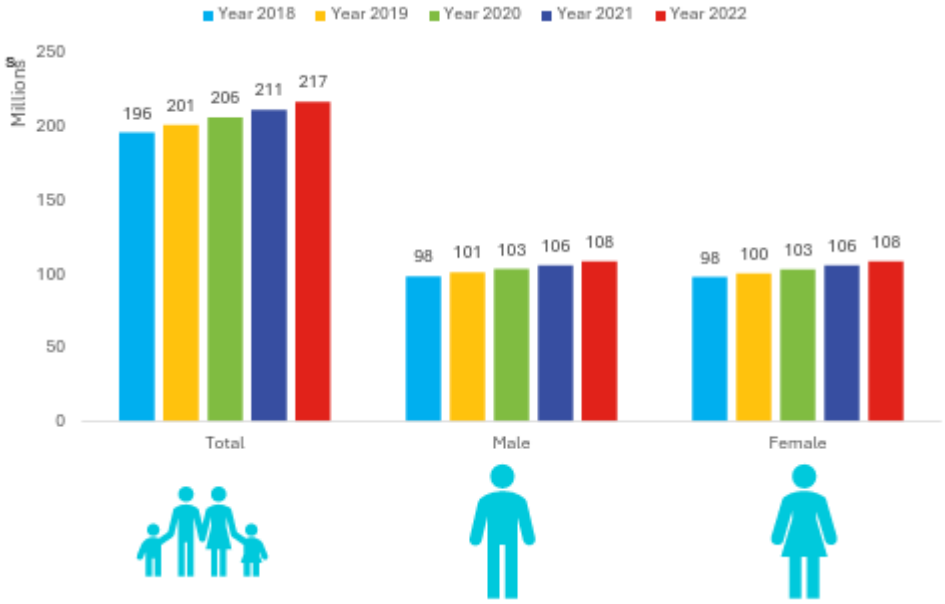


Figure 1: Nigeria Total Projected Population Distribution by Sex

Table 2: Model Variables

Variables	Variable description (population)	Unit	Baseline Value	Scenario Value	Source/References
S (t)	Skilled/eligible people remaining in the home country	persons	216,783,381	216,783,381	(Adeniji et al.,2024)
P (t)	People who have Secured employment in the home country	persons	42,500	42,500	(NBS,2023):0.85x50,00 slots
D (t)	Skilled emigrants residing outside the home country	persons	460,743	460,743	(Adeniji et al.,2024)
C (t)	Corruption-frustrated individuals in home country	persons	350,000	350,000	Asumed
U(t)	Unemployed people in the home –country labour force	persons	9,755,252.15	9,755,252.15	(NBS,2023):4.5% of S(0)

Variables	Variable description (population)	Unit	Baseline Value	Scenario Value	Source/References
R (t)	Returnees to home country	persons	13,917	13,917	(Adeniji et al.,2024)

The entire human population at time (t) is given by
$N(t) = S(t) + D(t) + P(t) + C(t) + U(t) + R(t)$,

Table 3: Model Parameters

Symbol	Baseline Value	Scenario Value	Parameter Description	Unit	Source/Reference
∧	8,086,020.11	8,086,020.11	Recruitment into S (0.0253+0.012) x 216,783,381.	Persons/year	(NBS,2023)
β	$2.198712676 \times 10^{-11}$	$2.198712676 \times 10^{-11}$	Coefficient of the S to D migration flow $\beta(1-\varepsilon)SD$	$Person^{-1}year^{-1}$	Assumed
α	2.8868 x 10^{-12}	2.8868×10^{-12}	Coefficient of the S to R flow αSP	$Person^{-1}year^{-1}$	Assumed
Ɛ	0.9	0.9	Talent-retention effectiveness	Dimensionless	Assumed
θ	1 x 10^{-7}	1 x 10^{-7}	Coefficient of the D to R flow θDP	$Person^{-1}year^{-1}$	Assumed
μ	0.012	0.012	Natural mortality rate	$year^{-1}$	Assumed
$\boldsymbol{r_o}$	0.03	0.03	Intrinsic growth rate in the P equation	$year^{-1}$	Assumed
$\boldsymbol{r_1}$	0.000193	0.000193	Depletion/utilization rate in the P equation	$year^{-1}$	(Adeniji el al.,2024)
γ	0.05	0.05	Rate coefficient for the R to S flow γR	$year^{-1}$	Assumed
K	1,259,750	1,259,750	Carrying capacity for P in logistic growth term	Persons	Assumed, based on (NBS,2023)
ρ	0.5	0.5	Rate of media awareness associated coefficient of the R to S transfer	$year^{-1}$	Assumed
M	0.003	0.003	Coefficient in the exponential factor e^{-MD}	$Person^{-1}$	Assumed
X	0.03	0.03	Migration influence rate	$year^{-1}$	Assumed
E	2.718281828	2.718281828	Natural exponential constant	Dimensionless	Mathematical Constant
φ	$1.758970141 \times 10^{-9}$	$1.758970141 \times 10^{-9}$	Effect of unemployment coefficient of the P to C interaction term	$person^{-1}year^{-1}$	Assumed
η	$1.319227606 \times 10^{-9}$	$1.319227606 \times 10^{-9}$	Anticipated rewards in the home country associated coefficient in the C to S	$person^{-1}year^{-1}$	Assumed
σ	$1.099356338 \times 10^{-9}$	$1.099356338 \times 10^{-9}$	Rate of Corruption	$year^{-1}$	Assumed
π	$1.319227606 \times 10^{-9}$	$1.319227606 \times 10^{-9}$	Influence rate of returnees on Unemployment	$person^{-1}year^{-1}$	Assumed
ψ	$1.978841 \times 1080^{-9}$	$1.978841 \times 1080^{-9}$	Employment Opportunity depletion rate	$person^{-1}year^{-1}$	Assumed

Table 4: Sensitivity of R_0

Parameters	Sensitivity Index
∧	+1.000000
β	+1.000000
α	-3.84x10^{-6}
Ɛ	-9.000000
θ	-0.640333
μ	-0.671244
r_o	-0.438458
r_1	+0.006940
γ	+3.4172 x10^{-7}
K	-0.640336
ρ	+3.42172 x10^{-6}
X	-0.256905

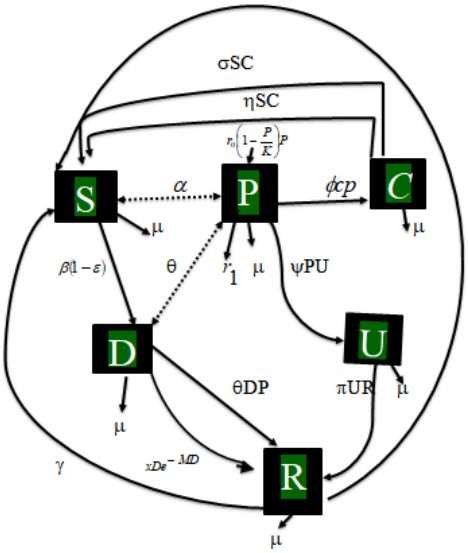


Figure 2: Schematic diagram of the improved six-compartment Japa model

Mathematical Analysis

To understand the mathematical characteristics of the new six compartment model we can study whether a solution exists, is unique and non-negative and determine the boundedness of the feasible region by using common ODE theorems (Birkhoff & Rota,1989). Next we find the steady states and compute the basic reproduction number using the next generation matrix method (Van den Driessche & Watmough,2002) Finally we explore the stability of our equilibria by using the given Jacobian and Lyapunov methods. Where applicable, the invariance principle is used by applying LaSalle's theorem (LaSalle,1960).

Non-Negativity of Solutions

$\Omega+= \{(S,P,C,U,D,R) \in R^6: S,P,C,U,D,R \geq 0\}$

Theorem: Suppose all parameters are non-negative and $0 \leq \varepsilon \leq 1$

For any initial condition

(S(0), P(0), C(0), U(0), D(0),R(0)) $\in \Omega_+$

Proof: The right-hand sides of system are locally lipschitz, so a unique local solution exists. To show that the solution cannot leaveΩ +. Evaluate each equation on the boundary where its corresponding compartment is zero.

$S = 0: \dot{S} = \wedge + (\gamma + \rho)R \geq 0$

$P = 0: \dot{P} = 0$

$C = 0: \dot{C} = 0$

$U = 0: \dot{U} = 0$

$D = 0: \dot{D} = 0$

$R = 0: \dot{R} = \theta DP + \alpha SP + xDe^{-MD} + \pi UR \geq O$

$R = 0: \dot{R} = ODP + \propto SP + xDe^{-MD} + \Lambda UR \geq 0$

Hence, (S(t), P(t), C(t), U(t), D(t), R(t)) $\in \Omega_+$

For all t in the solutions interval of existence.

Therefore, all compartments remain non-negative.□

Boundedness and Invariant Region

$N_h(t) = S(t) + P(t) + C(t) + U(t) + D(t) + R(t)$

Theorem: suppose the compartments and parameters are non-negative, K>0 and μ>0. For every non-negative initial condition, the solution remains bounded in the feasible region defined below.

Proof:

$\frac{dN_h}{dt} = \wedge + r_0 p(l - \frac{p}{k}) - r_1 p - \mu N_h$

For $p \geq 0$ and $r_0 \geq 0$

$r_0 p = (l - \frac{P}{K}) \leq \frac{rok}{4}, -r_1 p \leq 0$

Therefore, $\frac{dN_h}{dt} \leq \Lambda + \frac{r_0 k}{4} - \mu N_h$ Let $A = \Lambda + r_0\, {}^{k}/_{4}$. The differential inequality gives $N_h(t) \leq N_h(0)e^{-\mu t} + \frac{A}{\mu}(1 - e^{-\mu t}) \leq max\{N_h(0), \frac{A}{\mu}\}$ Thus, define $N\ max\{_h\ \frac{\Lambda + r_0\, {}^{k}/_{4}}{\mu}\ _{max}\}$ The p-equation can be written

$\frac{dp}{dt} = p[r_0(1 - \frac{p}{k}) - r_1 - \varphi c - \psi U - \mu]$

Because C, $U \geq 0$ and the relevant parameters are non-negative $\frac{dp}{dt} \leq r_0 p(1 - \frac{p}{k})$

Hence $p(t) \leq p_{max,}$ Where $p\ max\{_{max,}$ Consequently, the feasible region

$\Omega = \{(S, P, C, U, D, R) \in R_+{}^6 : P \leq P_{MAX,} S + P + C + U + D + R \leq N_{MAX}\}$

Is bounded.[Birkhoff and Rota 1989). □

Brain-Drain-Free Equilibrium

Let $q = \gamma + \rho + \mu$ At equilibrium, with $\overset{\cdot}{D} = \overset{\cdot}{C} = \overset{\cdot}{U} = 0,$ the p-equation becomes.

$0 = \overset{*}{p}[^{r_o(1 - \frac{\overset{\cdot}{p}}{k}) - r_1 - \mu}]$ For positive employment, $\cdot^*$ this gives

$\overset{\cdot}{p} = k(1 - \frac{r_1 + \mu}{r_o})$

$r_0 \geq r_1 + \mu$

The R-equation at this equilibrium gives

$0 = \alpha S^* P^* - qR^*, R^* = \frac{\alpha S^* P^*}{q}$

Using this expression in the S-equation,

$0 = \Lambda - \alpha S^* P^* - \mu S^* + (\gamma + \rho)R^*$

$S^* = \frac{\Lambda}{\mu(1 + \frac{\alpha P^*}{q})}, R^* = \frac{\alpha P^*}{q}\frac{\Lambda}{}$

$E_0{}^+ = \left(\frac{\Lambda}{\mu(1 + \frac{\alpha P^*}{q})}, p^*, 0,0,0, \frac{\alpha p^*}{q}\frac{\Lambda}{\mu(1 + \frac{\alpha p^*}{q})}\right), p^* = k(1 - \frac{r_1 + \mu}{r_0}$

If $r_0 \leq r_1 + \mu, p^* \leq 0, E_0{}^{(0)} = \left({}^{\Lambda}/_{\mu}, 0,0,0,0,0\right)$

Brain-Drain Endemic Equilibrium

At equilibrium, C=0 and C^*>0, SO $S^* = \frac{\varphi P^* - \mu}{\eta + \sigma}$ From D=0 with D^*>0

$\beta(1 - \varepsilon)S^* - \theta P^* - xe^{-MD} - \mu = 0>$

Which gives, $D^* = -\frac{1}{M} In\left(\frac{\beta(1-\varepsilon)S^* - \theta P^* - \mu}{x}\right)$ M>0, x>0 0< $\frac{\beta(1-\varepsilon)S^* - \theta P^* - \mu}{x}$ <1

D^*>0, U=0 with U^*>0

$R^* = \frac{\psi P^* - \mu}{\pi}$

$C^* = \frac{r_0(1 - \frac{p^*}{k}) - r_1 - \mu - \psi U^*}{\varphi}$

$U^* = \frac{(\gamma + \rho + \mu)R^* - \theta D^* P^* - \alpha S^* P^* - xD^* e^{-MD}}{\pi R^*}$

$0 = \Lambda - \beta(1 - \varepsilon)S^* D^* - \alpha S^* P^* - \mu S^* + (\gamma + \rho)R^* + (\eta + \sigma)S^* C^*$

$E^* = (S^*, P^*, C^*, U^*, D^*, R^*)$

Basic Reproduction: Number for Brain Drain

For the brain drain- free equilibrium, $D^* = C^* = U^* = 0$ While

$S^*, P^*, and\ R^*$ Positive

$E_0 = (S_0, P_0, 0,0,0, R_0)$

$D^* = \beta(1 - \varepsilon)SD - (\theta P + \mu)D - xDe^{-MD}$

$F = [\beta(1-\varepsilon)S_0], V = [\theta P_0 + x + \mu]$

$V^{-1} = \dfrac{1}{\theta P + x + \mu}$

$FV^{-1} = [\beta(1-\varepsilon)S_0][\dfrac{1}{\theta P_0 + x + \mu}]$

$\left[\dfrac{\beta(1-\varepsilon)S_0}{\theta P_0 + x + \mu}\right]$

$R_0 = \rho(FV^{-1}) = \dfrac{\beta(1-\varepsilon)S_0}{\theta P_0 + x + \mu}$

$p_0 = k(1 - \dfrac{r_1 + \mu}{r_0}), s_0 = \dfrac{\Lambda}{\left(\mu(1 + \dfrac{\alpha p_0}{\gamma + \rho + \mu}\right)}$

$$R_0 = \frac{\beta(1-\varepsilon)\Lambda}{\mu\left[1 + \dfrac{\alpha k\left(1 - \dfrac{r_1+\mu}{r_0}\right)}{\gamma + \rho + \mu}\right]\left[\theta k\left(1 - \dfrac{r_1+\mu}{r_0}\right) + x + \mu\right]}$$

We get
0.0126874<1,
R_0<1

Local Stability of the Brain-Drain-Free Equilibrium

$p_0 = k\left(1 - \dfrac{r_1+\mu}{r_0}\right), r_0 > r_1 + \mu, C^* = U^* = P^* = 0$

$S_0 = \dfrac{\Lambda}{\lambda\left(1 + \dfrac{\alpha P_0}{q + \mu}\right)}, R_{ret,0} = \dfrac{\alpha S_0 P_0}{q + \mu}$

Here $R_{ret,o}$ is the equilibrium number of returnees;
At this equilibrium, the jacobian in the order (S, P, C, U, D, R) is

$$\begin{pmatrix} -\alpha P_0 - \mu & -\alpha S_0 & (\eta+\sigma)S_0 & 0 & -\beta(1-\varepsilon)S_0 & q \\ 0 & \dfrac{-r_0 p_0}{k} & -\phi P_0 & -\psi P_0 & 0 & 0 \\ 0 & 0 & \phi P_0 - (\eta+\sigma)S_0 - \mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi P_0 - \pi R_{ret,0} - \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta(1-\varepsilon)S_0 - \theta P_0 - x - \mu & 0 \\ \alpha P_0 & \alpha S_0 & 0 & \pi R_{ret,0} & \theta P_0 + x & -(q+\mu) \end{pmatrix}$$

At D=0

$\dfrac{\partial}{\partial D}(xDe^{-MD}D = 0$=x s_0 $R_0 = \dfrac{\beta(1-\varepsilon)S_0}{\theta P_0 + x + \mu}$

The characteristics equation factors into six eigenvalue factors are

$\lambda_P = \dfrac{-r_0 p_0}{k}, \lambda_C = \varphi p_0 - (\eta + \sigma) - \mu, \lambda_U = \psi P_0 - \pi R_{ret,0} - \mu$

$\lambda_D = \beta(1-\varepsilon)S_0 - \theta P_0 - x - \mu = (\theta p_0 + x + \mu)(R_0 - 1)$

$\lambda_S - \mu, \lambda_R = -(\alpha P_0 + \gamma + \rho + \mu)$

The brain-drain-free equilibrium is locally asymptotically stable if
R_0<1, φP_0<$(\eta + \sigma)S_0 + \mu$, ψP_0<$\pi R_{ret,0} + \mu$
This under stated parameter mapping, the brain drain-free equilibrium is locally asymptotically stable

Table 5: The Eigenvalues

Mode	Eigenvalue $year^{-1}$
λ_C	-1.640405
λ_U	-0.010524
λ_D	-0.115293
λ_P	-0.017807
λ_S	-0.012000
λ_R	-0.562002

Global Stability of the Brain-Drain-Free Equilibrium

Let $E_0 = (S_0, P_0, 0, 0, 0, R_{RET,0}), P_0 = K(1 - \dfrac{r_1+\mu}{r_0}), s_0 = \dfrac{\Lambda}{\mu\left(1+\frac{\alpha p_0}{\gamma+\rho+\mu}\right)}, R_{ret,0} = \dfrac{\alpha s_0 p_0}{\gamma+\rho+\mu}$

Assume solutions lie in a bounded positively invariant region

So, with $0 \le S \le \bar{S}, 0 \le P \le \bar{P}, 0 \le D \le \bar{D}$

Hence $\bar{S}, \bar{P}, \bar{D}$ are symbolic upper bounds on that region.

Theorem:

Suppose $\delta_C = \mu - \varphi\bar{P} > 0$, $\delta_U = \mu - \psi\bar{P} > 0$, $\delta_D = \mu + xe^{-MD} - b\bar{S} > 0$
Define E=C+U+D
From the model equations
$\dot{E} = C(\varphi P - aS - \mu) + U(\psi P - \pi R - \mu) + D(bS - \theta P - xe^{-MD} - \mu)$
Since S, P, D ≥ 0, the assumed bounds give
$\dot{E} \leq -\delta_c C - \delta_U U - \delta_D D \leq -\delta E$
Where $\delta = min\{\delta_c, \delta_{u,} \delta_D\} > 0$.
Therefore, C(t), U(t), and D(t) tend to Zero.
For P>0, define
$W_P(P) = P - P^* - P^* In(\frac{p}{p^*})$
This function is non-negative and equals zero only when $p = p^*$using the p-equation,
$\dot{W}_P(P) = -\frac{r_o}{k}(p - p^*)^2 - (p - p^*)(\varphi C + \psi U)$
Let $Z = (S - S^*, R - R^*)^T$Where C=U=D=0 and P=P*,
The linear system for S and R has matrix
$A = \begin{pmatrix} -(\alpha P^* + \mu) & \gamma + \rho \\ \alpha P^* & -(\gamma + \rho + \mu) \end{pmatrix}$
Its trace is negative and its determinant is
$det(A) = \mu(q + \alpha p^*) > 0$
Thus, A is Hurwitz. Consequently, there exists a positive-define matrix H satisfying
$A^T H + HA = -I$
The equations for Z can be written as
$\dot{Z} = Az + f(t),$
Where the components of f(t), are
$f_1(t) = -bSD - \alpha S(P - P^*) + \alpha SC$
$F_2(t) = \alpha S(P - P^*) + \theta DP + xe^{-MD}D + \pi UR$
All variables are bounded in Ω. Hence, for some
Constant L>0.
$\|f(t) \| \leq L(E + |P - P^*|)$.
Set $W = W_P + Z^T H$ ƶ
The preceding relations imply that constants C>0 and L_1. >0 exist such that
$\dot{W} \leq -C((P - P^*)^2 + \|Z \|^2 + L_1 E\sqrt{(P - P)^2 + \|Z \|^2}.$
Now Consider the lyapunov function
V=E+Kw Where k>0 is chosen sufficiently small because E is bounded on Ω.
So that $\dot{V} \leq -\frac{\delta}{2}E - \frac{kc}{2}((p - p^*) + \|Z \|^2)$
For every state in Ω except $E_{0,}$Where $\dot{v} = 0$.Also, $V \geq 0$ and V=0 only at $E_{0,}$
Therefore E_0 is globally asymptotically stable relative to $\Omega \cap \{p(0)\} > 0$.

MATERIALS AND METHODS

Numerical Simulation Via Laplace Adomian Decomposition Method

We decompose the unknown compartments using the Adomian method by expressing them as infinite series along with expressing nonlinearities as Adomian polynomials (Adomian,1988) (Khuri,2001). These infinite series are substituted into the transformed equations resulting in recursive relations which are then solved iteratively. This allows us to find the approximate values of each compartment. In order to perform the numerical simulations we must restart the calculation using the LADM. This is done with increasing time intervals and the accuracy of our method can be analysed by using smaller steps and comparing to Radau IIA and LSODA.
Given the model solution via the application of Laplace Adomian Method
Consider the following system given by equation

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \beta(1 - \varepsilon)SD - \alpha SP - \mu S + \gamma R + \rho R + \eta SC + \sigma SC \\
\frac{dP}{dt} &= r_0(1 - \tfrac{P}{K})P - r_1 P - \varphi CP - \psi PU - \mu P \\
\frac{dC}{dt} &= \varphi CP - \eta SC - \sigma SC - \mu C \\
\frac{dU}{dt} &= \psi PU - \pi UR - \mu U \\
\frac{dD}{dt} &= \beta(1 - \varepsilon)SD - \theta DP - xDe^{-MD} - \mu D \\
\frac{dR}{dt} &= \theta DP + \alpha SP + xDe^{-MD} - \gamma R - \rho R - \mu R + \pi UR
\end{aligned} \qquad (2)$$

Take laplace of equation 1

$$\left.\begin{aligned}
L\{D^{\alpha}{}_{t}S(t)\} &= L\{\Lambda-\beta(1-\varepsilon)SD-\alpha SP-\mu S+\gamma R+\rho R+\eta SC+\sigma SC\}\\
L\{D^{\alpha}{}_{t}P(t)\} &= L\{r_0(1-\tfrac{P}{K})P-r_1P-\varphi CP-\psi PU-\ \mu P\}\\
L\{D^{\alpha}{}_{t}C(t)\} &= L\{\varphi CP-\eta SC-\sigma SC-\mu C\}\\
L\{D^{\alpha}{}_{t}U(t) &= L\{\psi PU-\pi UR-\mu U\}\\
L\{D^{\alpha}{}_{t}D(t)\} &= L\{\beta(1-\varepsilon)SD-\theta DP-xDe^{-MD}-\mu D\}\\
L\{D^{\alpha}{}_{t}R(t)\} &= L\{\theta DP+\alpha SP+xDe^{-MD}+\lambda UR-\gamma R-\rho R-\mu R\}
\end{aligned}\right| \quad (3)$$

$$S^{\alpha}S(t) = S^{\alpha-1}S(0)+L\{\Lambda-\beta(1-\varepsilon)SD-\alpha SP-\mu S+\gamma R+\rho R+\eta SC+\sigma SC\}$$
$$S^{\alpha}P(t) = S^{\alpha-1}P(0)+L\{r_0(1-\frac{P}{K})P-r_1P-\varphi CP-\psi PU-\ \mu P\}$$
$$S^{\alpha}C(t) = S^{\alpha-1}C(0)+L\{\varphi CP-\eta SC-\sigma SC-\mu C\}$$
$$S^{\alpha}U(t) = S^{\alpha-1}U(0)+L\{\psi PU-\pi UR-\mu U\}$$
$$S^{\alpha}D(t) = S^{\alpha-1}D(0)+L\{\beta(1-\varepsilon)SD-\theta DP-xDe^{-MD}-\mu D\}$$
$$S^{\alpha}R(t) = S^{\alpha-1}R(0)+L\{\theta DP+\alpha SP+xDe^{-MD}+\lambda UR-\gamma R-\rho R-\mu R\}$$
$$S(t) = S^{-1}S(0)+\tfrac{1}{S^{\alpha}}L\{\Lambda-\beta(1-\varepsilon)SD-\alpha SP-\mu S+\gamma R+\rho R+\eta SC+\sigma SC\}$$
$$P(t) = S^{-1}P(0)+\frac{1}{S^{\alpha}}L\{r_0(1-\frac{P}{K})P-r_1P-\varphi CP-\psi PU-\ \mu P\}$$
$$C(t) = S^{-1}C(0)+\frac{1}{S^{\alpha}}L\{\varphi CP-\eta SC-\sigma SC-\mu C\}$$
$$U(t) = S^{-1}U(0)+\frac{1}{S^{\alpha}}L\{\psi PU-\pi UR-\mu U\}$$
$$D(t) = S^{-1}D(0)+\frac{1}{S^{\alpha}}L\{\beta(1-\varepsilon)SD-\theta DP-xDe^{-MD}-\mu D\}$$
$$R(t) = S^{-1}R(0)+\frac{1}{S^{\alpha}}L\{\theta DP+\alpha SP+xDe^{-MD}-\gamma R-\rho R-\mu R\}$$

We can let $S(t)$,$P(t)$,$C(t)$,U(t),$D(t)$ and $R(t)$ be in the form of infinite series;
$S(t)=\sum_{i=0}^{k}S_i$,$P(t)=\sum_{i=0}^{k}P_i$,$C(t)=\sum_{i=0}^{k}C_i$,$U(t)=\sum_{i=0}^{k}U_i$ $D(t)=\sum_{i=0}^{k}D_i$and $R(t)=\sum_{i=0}^{k}R_i$
The non-linear term involved in the model are
$S(t)D(t)$,$S(t)P(t)$,$S(t)C(t)$,$C(t)P(t)$,$P(t)U(t)$,$U(t)R(t)$,$D(t)P(t)$
And it can be decomposed by adomian below, Let;
$S(t)D(t)=\sum_{i=0}^{k}W_i$,$S(t)P(t)=\sum_{i=0}^{k}Z_i$,$S(t)C(t)=\sum_{i=0}^{k}Q_i$,$C(t)P(t)=\sum_{i=0}^{k}M_i$,$P(t)U(t)=\sum_{i=0}^{k}A_i$,$U(t)R(t)=\sum_{i=0}^{k}B_i$and $D(t)P(t)=\sum_{i=0}^{k}F_i$
Where W_i,Z_i,Q_i,M_i,A_i,B_i, and F_i are adomian polynomial given by

$$W_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x S_x\sum_{x=0}^{i}\Lambda^i D_x]\lambda$$
$$Z_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x S_x\sum_{x=0}^{i}\Lambda^i P_x]\lambda$$
$$Q_i=\tfrac{1}{\Gamma(i+1)}\tfrac{di}{dt}[\textstyle\sum_{x=0}^{i}\Lambda^x S_x\sum_{x=0}^{i}\Lambda^i C_x]\lambda$$
$$M_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x C_x\sum_{x=0}^{i}\Lambda^i P_x]\lambda$$
$$A_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x P_x\sum_{x=0}^{i}\Lambda^i U_x]\lambda$$
$$B_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x U_x\sum_{x=0}^{i}\Lambda^i R_x]\lambda$$
$$F_i=\frac{1}{\Gamma(i+1)}\frac{di}{dt}[\sum_{x=0}^{i}\Lambda^x D_x\sum_{x=0}^{i}\Lambda^i P_x]\lambda$$

By Substituting we get

$$S(t) = S^{-1}S(0)+\frac{1}{S^{\alpha}}L\{\Lambda-\beta(1-\varepsilon)W_i-\alpha Z_i-\mu S+\gamma R+\rho R+\eta Q_i+\sigma Q_i\}$$
$$P(t) = S^{-1}P(0)+\frac{1}{S^{\alpha}}L\{r_0(1-\frac{P}{K})P-r_1P-\varphi M_i-\psi A_i-\ \mu P\}$$
$$C(t) = S^{-1}C(0)+\frac{1}{S^{\alpha}}L\{\varphi M_i-\eta Q_i-\sigma Q_i-\mu C\}$$
$$U(t) = S^{-1}U(0)+\frac{1}{S^{\alpha}}L\{\psi A_i-\pi B_i-\mu U\}$$
$$D(t) = S^{-1}D(0)+\frac{1}{S^{\alpha}}L\{\beta(1-\varepsilon)W_i-\theta F_i-xDe^{-MD}-\mu D\}$$

$$R(t) = S^{-1}R(0) + \frac{1}{S^{\alpha}}L\{\theta F_i + \alpha Z_i + xDe^{-MD} - \gamma R - \rho R - \mu R\}$$

By using the initial conditions

$S(0) = \theta_1, P(0) = \theta_2, C(0) = \theta_3$, U(0)= $\theta_4, D(0) = \theta_5, R(0) = \theta_6$

We have

$$\left.\begin{aligned}
S(t) &= \frac{\theta_1}{S} + \frac{1}{S^{\alpha}}L\{\Lambda - \beta(1-\varepsilon)W_i - \alpha Z_i - \mu S + \gamma R + \rho R + \eta Q_i + \sigma Q_i\}\\
P(t) &= \frac{\theta_2}{S} + \frac{1}{S^{\alpha}}L\{r_0(1-\frac{P}{K})P - r_1 P - \varphi M_i - \psi A_i - \mu P\}\\
C(t) &= \frac{\theta_3}{S} + \frac{1}{S^{\alpha}}L\{\varphi M_i - \eta Q_i - \sigma Q_i - \mu C\}\\
U(t) &= \frac{\theta_4}{S} + \frac{1}{S^{\alpha}}L\{\psi A_i - \pi B_i - \mu U\}\\
D(t) &= \frac{\theta_5}{S} + \frac{1}{S^{\alpha}}L\{\beta(1-\varepsilon)W_i - \theta F_i - xDe^{-MD} - \mu D\}\\
R(t) &= \frac{\theta_6}{S} + \frac{1}{S^{\alpha}}L\{\theta F_i + \alpha Z_i + xDe^{-MD} - \gamma R - \rho R - \mu R\}
\end{aligned}\right| \quad (4)$$

Take Iteration each term in eq. (3) to get the solution of each compartment and taking the laplace inverse gives formula for the model

$$\left.\begin{aligned}
\sum_{i=0}^{\infty} S_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{\Lambda - \beta(1-\varepsilon)W_i - \alpha Z_i - \mu S + \gamma R + \rho R + \eta Q_i + \sigma Q_i\}]\\
\sum_{i=0}^{\infty} P_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{r_0(1-\frac{P}{K})P - r_1 P - \varphi M_i - \psi A_i - \mu P\}]\\
\sum_{i=0}^{\infty} C_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{\varphi M_i - \eta Q_i - \sigma Q_i - \mu C\}]\\
\sum_{i=0}^{\infty} U_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{\psi A_i - \pi B_i - \mu U\}]\\
\sum_{i=0}^{\infty} D_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{\beta(1-\varepsilon)W_i - \theta F_i - xDe^{-MD} - \mu D\}]\\
\sum_{i=0}^{\infty} R_{i+1}(t) &= L^{-1}[\frac{1}{S^{\alpha}}L\{\theta F_i + \alpha Z_i + xDe^{-MD} - \gamma R - \rho R - \mu R\}]
\end{aligned}\right| \quad (5)$$

Solving (4) by mathematica 11 software package, the followings were obtained;

When i=0

$$\left.\begin{aligned}
S_1 &= \frac{-t^{\alpha}(\lambda+\theta_1(-\mu-\alpha\theta_2+(\eta+\sigma)\theta_3+(-1+\varepsilon)\theta_5+(\gamma+\rho)\theta_6}{\Gamma(1+\alpha)}\\
P_1 &= \frac{-t^{\alpha}\theta_2((-1+k)r\theta-k(r_1+\mu)-k(\varphi\theta_3+\psi\theta_4}{\Gamma(1+\alpha)}\\
C_1 &= \frac{-t^{\alpha}(\mu+(\eta+\sigma)\theta_1-\varphi\theta_2)\theta_3}{\Gamma(1+\alpha)}\\
U_1 &= \frac{-t^{\alpha}\theta_4(\mu-\psi\theta_2+\pi\theta_6)}{\Gamma(1+\alpha)}\\
D_1 &= \frac{-t^{\alpha}xe^{-MD}(xe^{MD}(\mu+\beta(-1+\varepsilon)\theta_1+2\theta\theta_2)\theta_5+\theta_6}{\Gamma(1+\alpha)}\\
R_1 &= \frac{-t^{\alpha}xe^{-MD}(\theta_6+xe^{MD}(\theta_2(\alpha\theta_1+\theta\theta_5)-(\gamma+\mu+\rho-\pi\theta_4)\theta_6}{\Gamma(1+\alpha)}
\end{aligned}\right| \quad (6)$$

When, i=1

$$\left.\begin{aligned}
S_2 &= t^{\alpha}[\frac{\lambda}{\Gamma(1+\alpha)} + \frac{t^{\alpha}\lambda\mu}{\Gamma(1+2\alpha)} + \frac{t^{\alpha}xe^{-MD}}{K\Gamma(1+2\alpha)}(K(\gamma+\rho+(\beta_\beta\varepsilon)\theta_1)\theta_6 - xe^{md}[k{\theta_1}^2((\eta+\sigma)^2\theta_3 +\\
&\beta^2(-1+\varepsilon)^2\theta_5 + \theta_1(k\alpha^2{\theta_2}^2 + \theta_1(k\alpha^2\theta_2^2 + \theta_2(-k(2\alpha(\eta+\sigma) + (\alpha+\eta+\sigma)\theta_3 + \alpha(p-k(\square r_1+\gamma-\mu+\rho)\\
&-k\psi\theta_4) - 2k\beta(-1+\varepsilon)(\alpha-\theta)\theta_5) + k(\mu^2 + ((\eta+\sigma)\theta_3 + \beta(-1+\varepsilon)\theta_5)(-\mu + (\eta+\sigma)\theta_3 + \beta(-1+\varepsilon)\theta_5 + k(\lambda(\eta+\sigma)\\
&\theta_3 + \beta(-1+\varepsilon)\lambda\theta_5 + (\gamma+\rho)(\gamma+\rho+(\eta+\sigma)\theta_3 - \pi\theta_4 + \beta(-1+\varepsilon)\theta_5)\theta_6 - \theta_2(\alpha\pi + (\gamma+\rho)(\theta\theta_5 + \alpha\theta_6)))])]\\
P_2 &= \frac{t^{2\alpha}\theta_2}{k_3}[-4^{\alpha}r_o t^{\alpha}\Gamma({}^{1}/_{2} + \alpha)\theta_2(r_o - kr_o + k(r_1+\mu) + K\varphi\theta_3 + K\psi\theta_3)^2 + {}^{K^2}/_{\Gamma(1+2\alpha)}((r_0-r_1)((-1+k)r_o\\
&-k(r_1+\mu)) + \theta\theta_3(r_o - 2kr_o + 2k(r_1+\mu) + k(\eta+\sigma)\theta_1 + k\varphi(-\theta_2+\theta_2)) + k\psi(-r_o + r_1 + \varphi\theta_3)\theta_4)]\\
C_2 &= \frac{t^{2\alpha}\theta_3}{k\Gamma(1+2\alpha)}[k(\eta+\sigma)^2{\theta_1}^2 + k\varphi^2{\theta^2}_2 - \varphi\theta_2(r_o - kr_o + kr_1 + 3k\mu + k\varphi\theta_3 + k\varphi\theta_4) + k(\eta+\sigma)\theta_1(\mu -\\
&(\alpha+2\varphi)\theta_2 + (\eta+\sigma)\theta_3 + \beta(-1+\varepsilon)\theta_5) + k(\mu^2 + \lambda(\eta+\sigma) + (\gamma+\rho)(\eta+\sigma)\theta_6)]\\
U_2 &= \frac{t^{2\alpha}xe^{-MD}}{K\Gamma(1+2\alpha)}[k\pi\theta_4\theta_6 + xe^{MD}(k\sigma(\eta+\sigma)\theta_1{\theta_3}^2 + k\theta_3(-\mu^2 + \pi\sigma + \sigma(\eta+\sigma){\theta_1}^2 + \varphi\theta_2(\mu+\psi\theta_4) + \theta_1\\
&(-\mu(\eta+\sigma) - \sigma(\alpha+\varphi)\theta_2 + \beta(-1+\varepsilon)\sigma\theta_5) + (\gamma+\rho)\sigma\theta_6) + \theta_4(-k\psi\psi{\theta_2}^2 - k\pi\theta_6(\gamma+2\mu+\rho-\pi\theta_4+\pi\theta_6) +\\
&\theta_2((r_o - kr_o + kr_1 + 2k\mu)\psi + k(\pi\alpha\theta_1 + \psi\psi\theta_4 + \pi\theta\theta_5 + \pi(\psi+\psi)\theta_6))))]\\
D_2 &= \frac{t^{2\alpha}xe^{2MD}}{K\Gamma(1+2\alpha)}[kxe^{2MD}\beta^2(-1+\varepsilon)^2{\theta_1}^2\theta_5 + k(-1+xe^{MD}(\gamma+2\mu+\rho+2\theta\theta_2-\pi\theta_4))\theta_6 +\\
&kxe^{MD}\theta_1(-\alpha\theta_2 + xe^{MD}\beta(-1+\varepsilon)\theta_5(\mu - (\alpha-4\theta)\theta_2 + (\eta+\sigma)\theta_3 + \beta(-1+\varepsilon)\theta_5) - \beta\theta_6 + xe^{MD}\theta_5(-K\theta\theta_2 +\\
&xe^{MD}(K(\beta(-1+\varepsilon)\lambda + \mu^2) + 2\theta\theta_2(r_o - kr_o + kr_1 + 3k\mu + 2k\theta\theta_2\\
R_2 &= \frac{t^{2\alpha}xe^{-2MD}}{K\Gamma(1+2\alpha)}[kxe^{-2MD}{\theta_2}^2(\alpha^2\theta_1 - 2\theta^2\theta_5) + k(-1+xe^{MD}(\gamma+\mu+\rho))(-1+xe^{MD}(\gamma+\mu+\rho-\pi\theta_4))\\
&\theta_6 - xe^{MD}\theta_2(\theta_1[-k\alpha + xe^{MD}(\alpha(r_o - kr_o + k(r_1+\gamma+\mu+\rho) + k(\eta+\sigma+\varphi)\theta_3 + k\psi\theta_4) + k\beta(-1+\varepsilon)(\alpha+\theta)\theta_5)] + k\theta(-\theta_5+\theta_6]\\
&+xe^{MD}(\theta(r_o - kr_o + k(r_1+\gamma+3\mu+\rho) + k\varphi\theta_3 + k\psi\theta_4)\theta_5 + k\alpha(\lambda + (\gamma+\rho)\theta_6)))]
\end{aligned}\right|$$

Table 6: Step-Refinement and Solver-Validation Results for the Restarted Two-Correction LADM

LADM corrections	Maximum step (Years)	Restart steps	Maximum Scaled Error:LADM vs Radau	Maximum scaled Difference:Radau vs LSODA
i= 0,1	0.005	2,000	$8.20x10^{-6}$	$1.82x10^{-11}$
i= 0,1	0.0025	4,000	$2.05x10^{-6}$	$1.82x10^{-11}$
i= 0,1	0.001	10,000	$3.27x10^{-7}$	$1.82x10^{-11}$
i= 0,1	0.0005	20,000	$8.19x10^{-8}$	$1.82x10^{-11}$

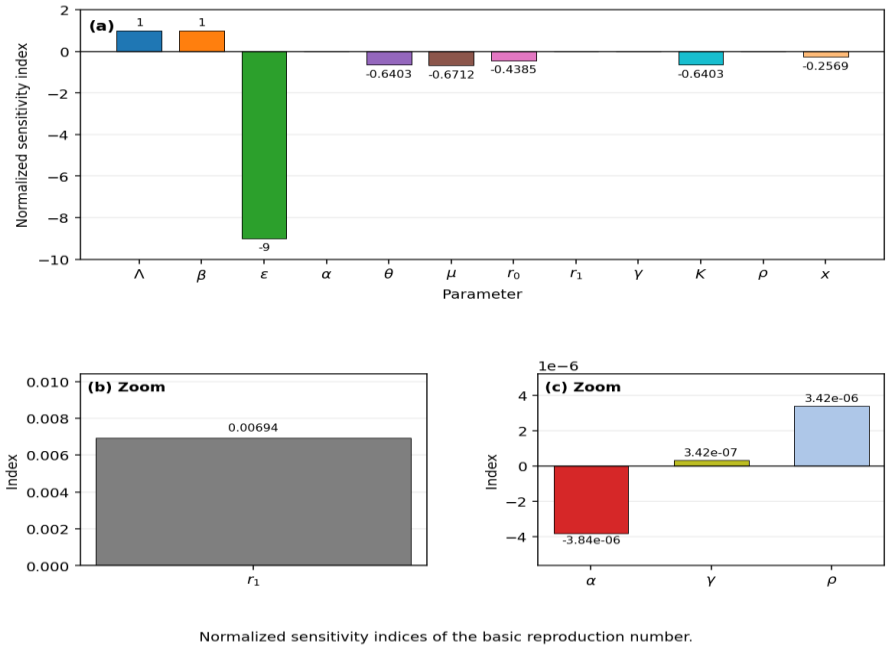


Figure 3: Normalized sensitivity indices of the basic reproduction number

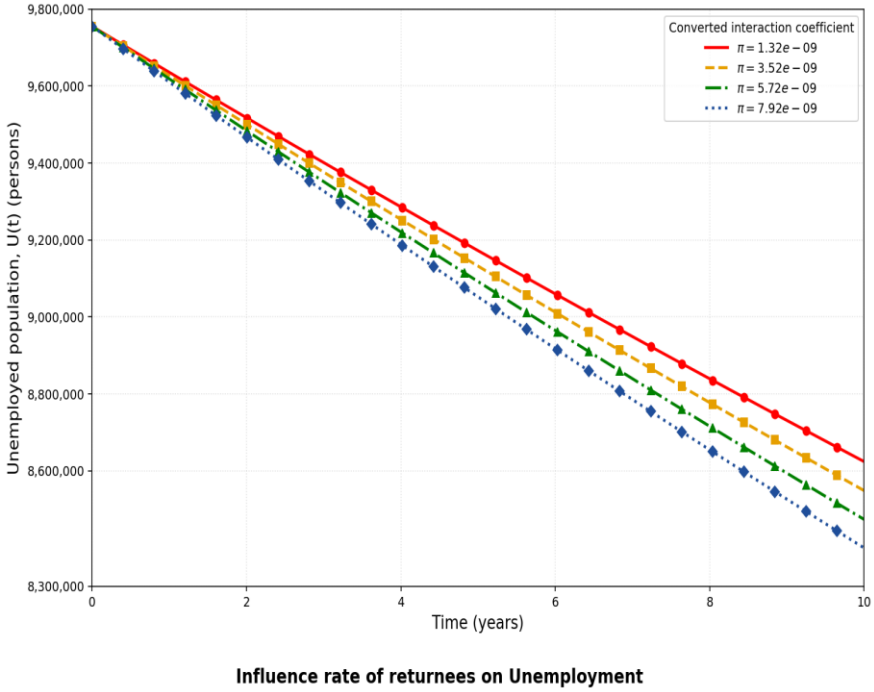


Figure 4: Influence rate of returnees on Unemployment

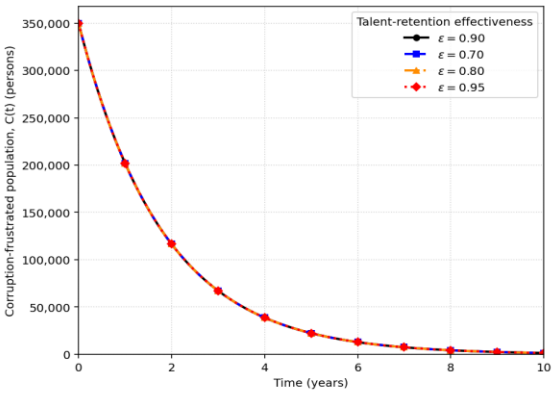


Figure 5: Corruption-frustracted population under varying talent retention

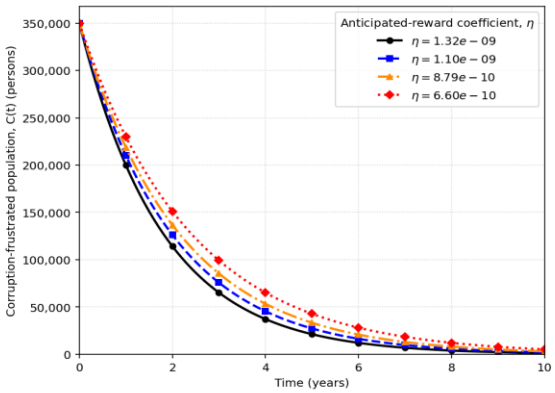


Figure 6: Corruption-frustracted population under varying anticipated reward

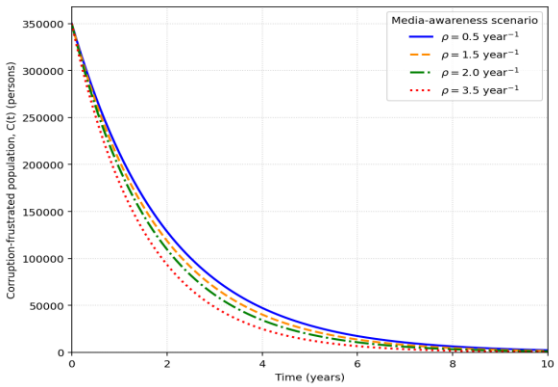


Figure 7: Effect of media awareness on the corruption-frustrated population

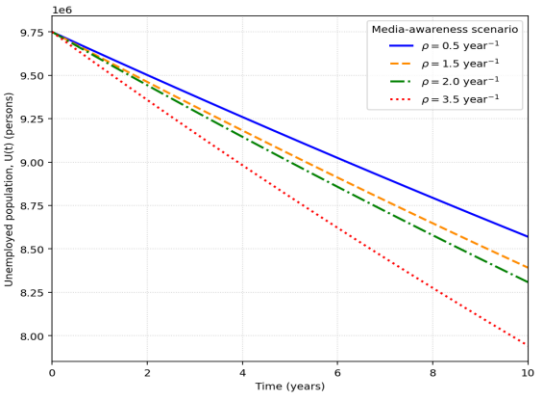


Figure 8: Effect of media awareness on unemployment

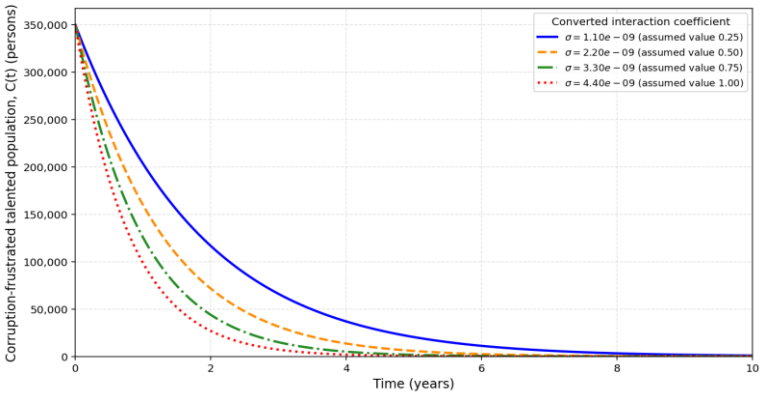


Figure 9: Corruption-frustrated talented population under alternative of the C to S Interaction coefficient sigma

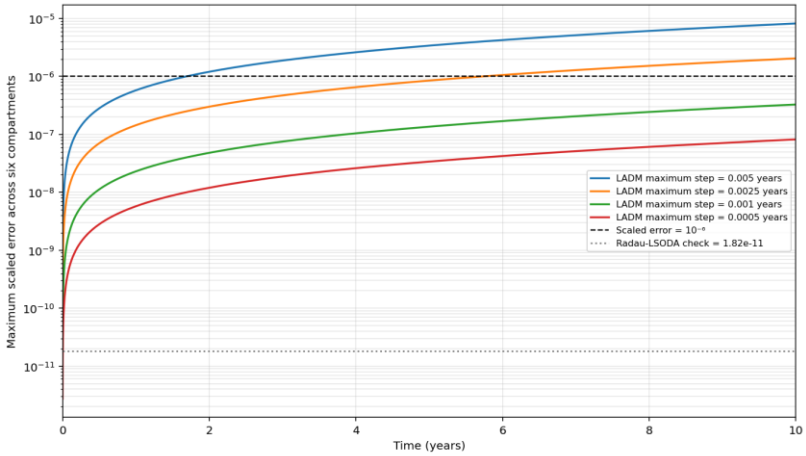


Figure 10: Error and step-refinement assessment of restarted two-correction LADM relative to RadauIIA

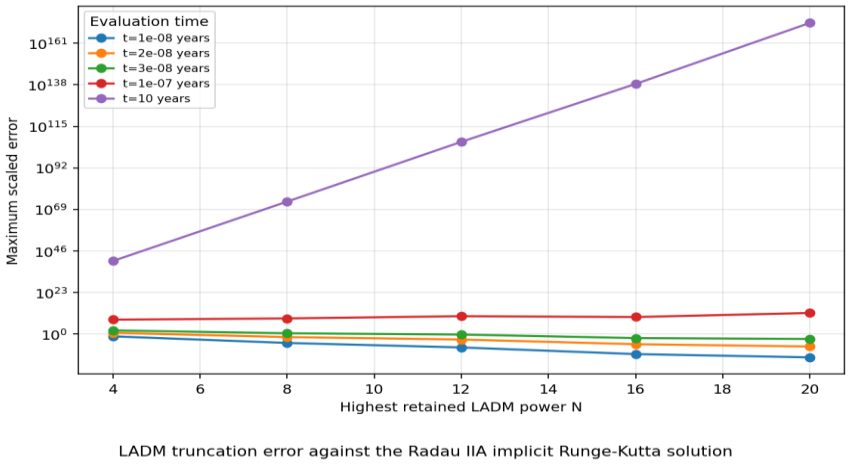


Figure 11: LADM truncation error against the Radau IIA implicit Runge-Kutta solution

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RESULTS AND DISCUSSION

The numerical graphs portray the evolution of the unemployed population, U(t), and of the corruption-frustrated population, C(t), over a ten-year period and for different parameter values; the curves remain non-negative throughout the displayed interval. These are model scenario results and as such are illustrative of the model's

Basic Reproduction Number and Sensitivity Analysis

With the updated base case parameter values, the basic reproduction number is equal to $R_0 = 0.0126874$. As this value is less than one, it satisfies the threshold condition for local asymptotic stability of the brain drain free equilibrium.

Figure 2 displays the sensitivity indices of R 0. The sensitivity index for the talent retention effectiveness is ε = -9 whereas both recruitment rate Λ and emigration rate β have sensitivity indices of +1. As the sensitivity index is negative, we see that the R 0 will decrease as we increase the effectiveness of talent retention. From this we see that talent retention and domestic recruitment would be key areas to look into.

Population Scenarios

Figure 3 shows the unemployment levels decrease over time for the different returnee rate scenarios. As the returnee rate value increases the curves diverge.

Figure 4 shows the corruption frustrated population decreasing for the different talent retention scenarios. The curves are very similar.

Figure 5 shows the corruption frustrated population decreasing more quickly for higher anticipated reward values.

Figure 6 shows the corruption frustrated population decreasing for the different media awareness scenarios. The curves for the higher media awareness scenarios decrease more quickly. Figure 7 shows the unemployment levels also decrease for the different media awareness scenarios.

Figure 8 shows the corruption frustrated talented population decreasing more quickly for higher values of σ. This is due to individuals being transferred from the corruption frustrated population C to the source country talent pool S based on the interactions.

Figures 3-8 show how the model can be used to compare different scenarios. This could help government and policy researchers understand what methods they could look into further, such as improving domestic opportunities.

Numerical Analysis

Figure 9 shows how the maximum scaled error decreases as the maximum step size is decreased. The maximum scaled error decreases from 8.20 × 10 -6 when the step size was 0.005 years to 8.19 × 10 -8 when the step size was 0.0005 years. This is approximately a hundred times smaller. The

maximum scaled difference between the Radau IIA and LSODA is approximately 1.82 × 10 -11 which suggests that both solutions are similar.
Figure 10 shows how the truncation error for the LADM method compared to the Radau IIA solution behaves over the time period. This shows how important it is to manage the accumulation of the truncation error. When combined with the information from Figure 9 we can see that the restarted and step refinement LADM method can be used.

Conclusion

This study created a 6 compartment mathematical model to represent brain drain dynamics. The model includes the source country population, employed population, corruption frustrated population, unemployed population, emigrants and returnees. This model builds on the work of Adeniji et al (2024) by including additional factors such as unemployment, corruption frustration, anticipated rewards, talent retention and media awareness. The model was analysed to investigate its non-negative properties, boundedness, steady state behaviour, basic reproduction number, sensitivity and numerical solution.
With the updated base case parameters the basic reproduction number becomes $R_0 = 0.0126874$ which is less than one. This means that the threshold condition for local asymptotic stability of the brain drain free equilibrium is satisfied. The sensitivity analysis shows that the talent retention effectiveness is an important factor.
The scenarios show how increasing/decreasing talent retention, anticipated rewards, media awareness, returnee rates and the interaction coefficient between C and S will affect the population levels within the model. This can be seen by looking at the curves in the figures where the corruption frustrated population and unemployment levels reduce over time. The Radau IIA and LSODA solutions suggest that both solutions are similar. Future work can be done to calibrate the assumed parameters using suitable data and compare the model against real population trends.

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